ON THE WELL – POSEDNESS OF BACK PROPAGATION ALGORITHMS IN INVERSE ELECTROMAGNETICS

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Approximate backpropagation (ABP) methods have been used by the author to identify the shape of simple scatterers in the resonance region from full aperture data both in the acoustic and electromagnetic case. ABP methods rely on a relation between the expansion coefficients, which represent the scattered wave in the far zone and, respectively, on the obstacle boundary, Γ, and lead to minimization algorithms.

In spite of satisfactory computational results, the well–posedness of ABP generally remains an open problem. A related result, which may eventually justify the method, pertains to a ***forward*** propagator i.e., to the affine map $\mathbf{N}[.] := = \Re\mathbf{L}[.] + \mathbf{b}$, which appears in the solution of the direct scattering problem.

Very briefly, if both the scattered wave and its normal derivative on Γ are represented by uniformly converging series according to a suitable basis in $L^2(\Gamma)$, if $\mathbf{N}[.]$ is bounded and maps ℓ_2 sequences of scattering coefficients into ℓ_2 sequences and if the spectral radius $r_\sigma[\Re\mathbf{L}]$ of $\Re\mathbf{L}$ satisfies $r_\sigma[\Re\mathbf{L}] < 1$, then $\forall\, \mathbf{b} \in \ell_2$ there exists a unique fixed point, $\mathbf{f}$, for the map $\mathbf{p}[t+1] = \mathbf{b} + \Re\mathbf{L} \cdot \mathbf{p}[t]$, $t = 0, 1, 2, \ldots$, obtained by successive approximations and started with an arbitrary $\mathbf{p}[0] \in \ell_2$.

One then derives the convergence and consistency properties of the approximate forward propagator, which relates finite subsequences of coefficients.

These results are applied to a class of numerical problems, which include the inversion of the IPSWICH Data.