

On the Well-Posedness of Propagation Algorithms in Inverse Electromagnetics

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THIS TALK WANTS TO BE
A TRIBUTE TO THE MEMORY OF
RALPH E KLEINMAN
A TEACHER,
A GUIDE,
AND AN EXAMPLE
OF FULL COMMITTMENT TO RESEARCH.

PLAN

Statement of the inverse electromagnetic problem

Spatial dimension: $n = 2$.

Data: IPSWICH {RCS, χ }.

Prior knowledge: {V polarization; $\{\mathbf{k}_q^{(\text{inc})}\}$; DIRICHLET BC on $\partial\Omega$; Ψ_{adm} }.

Parameters: {obstacle; scattered field approximation} = $\{I; L\}$.

Unknowns: {shape parameters; phase offsets} = $\{\vec{\psi}; \gamma\}$.

Top – down heuristics of shape reconstruction:

minimization of the far zone defect
(*approximate forward propagation*, AFP).

Numerical results:

full vs. limited aperture data,
superposition of incident waves.

Bottom – up justification of the algorithm: an attempt

The *least squares* boundary coefficients, $\mathbf{c}^{(L)}$:

- * error bounds;
- * RAYLEIGH' s hypothesis.

Approximate forward propagation

- * successive approximations and fixed points;
- * convergence, trust and effectiveness;
- * the spectral radius of a propagator matrix.

Further problems.

INVERSION SCHEMES FOR VERTICAL POLARIZATION AND PERFECTLY ELECTRICAL CONDUCTING, STAR SHAPED OBSTACLE, $n = 2$

Obstacle: prior knowledge and parameterization

$$\begin{aligned} \Gamma := & \{ \mathbf{x} \mid r = r[\phi], 0 \leq \phi \leq 2\pi, r[\cdot] \in C^2([0, 2\pi]), \\ & 0 < r_{\min} \leq r[\phi] \leq r_{\max}, \forall \phi \in [0, 2\pi] \} \\ \frac{1}{r^2}[\phi] = & \psi_1 + \psi_2 \cos \phi + \psi_3 \sin \phi + \psi_4 \cos 2\phi + \dots \text{ (I terms) } \\ \Rightarrow \{ \text{constraints on } r[\cdot] \} \Leftrightarrow \{ \vec{\psi} \in \Psi_{\text{adm}} \}. \end{aligned}$$

Data

incident waves

$$\{ \mathbf{E}_q^{(\text{inc})} = \hat{\mathbf{z}} e^{i \mathbf{k}_q \cdot \mathbf{r}}, q \in Q \}$$

RCSS (σ_q) and phases (χ_q)

$$\left\{ \frac{\sigma_q[\phi_j]}{\text{wavelength}}; \chi_q[\phi_j; \gamma_q] \right\}$$

$$A[\mathbf{k}_q; \phi_j; \gamma_q]$$

scattering amplitude

Scattered waves $\{ \mathbf{E}_q^{(\text{sc})} \}$

$$\hat{\mathbf{z}}(\Delta + k^2)E_{q,z}^{(\text{sc})} = \mathbf{0} \text{ in } \mathbb{R}^n \setminus \overline{\Omega} ; (E_{q,z}^{(\text{inc})} + E_{q,z}^{(\text{sc})})|_{\Gamma} = 0 ; \text{ SRC}$$

Approximation of the scattered waves

... in the far zone ...

from $E_{q,z}^{(\text{sc})}[\mathbf{x}] = \sum_{\lambda} f_{q,\lambda} v_{\lambda}[\mathbf{x}], \mathbf{x} = \{r, \phi\}; \lambda = \{p, l\}; p = 0, 1; p \leq l,$

where $v_{\lambda} := \sqrt{\epsilon_l} H_l^{(1)}[kr] [(1-p)\cos l\phi + p\sin l\phi],$

to $\Lambda[L] := \{p, l \mid p = 0, 1; p \leq l \leq L\}$

and $E_{q,z}^{(\text{sc}, L)}[\mathbf{x}] := \sum_{\lambda \in \Lambda(L)} p_{q,\lambda}^{(L)} v_{\lambda}[\mathbf{x}],$ with $\{p_{q,\lambda}^{(L)}\}$ suitable

... on the obstacle boundary ...

$$E_{q,z}^{(\text{sc}, L)}|_{\Gamma} := \sum_{\lambda \in \Lambda(L)} c_{q,\lambda}^{(L)} v_{\lambda}|_{\Gamma}, \text{ with } \{c_{q,\lambda}^{(L)}\} \text{ suitable.}$$

TOP – DOWN HEURISTICS:

SHAPE RECONSTRUCTION BY $F_{DV}^{(L)}$ MINIMIZATION

PBM (shape reconstruction from card[Q] sets of IPSWICH data)

Given $I, L, \{A[\mathbf{k}_q; \phi_j; \gamma_q], j \in N_q, q \in Q\}$, find $\vec{\psi}^\star \in \Psi_{\text{adm}}$ and $\vec{\gamma}^\star$ such that,

$$F_{DV}^{(L)}[\vec{\psi}^\star; \vec{\gamma}^\star] = \min$$

where

