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Banking and monetary policy in a monetary union

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ABSTRACT

We model the banking systems of a two-country monetary area with a single central bank. Banks can operate either locally or in national loan markets and participate in an area-wide interbank market. This model determines open-market and retail bank interest rates and equilibrium allocations of the banks' assets and liabilities in relation to the exogenous policy choices of the central bank and its subsequent balance sheet allocations. We find that loan and deposits markets are interdependent and that regional shocks generate inter-regional spillovers. The central bank of our model has several alternative tools to provide liquidity and to influence all the short-term interest rates of the monetary area. We analyze the effects of various alternative central bank policy instruments in both scarce and abundant-reserves regimes.

1. Introduction

In large economies like those of the United States, the European Union, or China, the banking industry is segmented across regions. In U.S. or European states and Chinese provinces, most lending activities are conducted by local players, while only a handful of very large intermediaries operate on a national level. In these large economies, regional characteristics matter, both because a number of institutional features are different across regions but also because both the level of economic development and the business cycles dynamics display large variations.[2] In spite of the large structural and cyclical differences, each of these economies is characterized by a common set of institutions, a common currency, and a common monetary policy. The presence of a well-functioning common interbank market allows banks operating in different regions to interact, and via this market any excess of deposits over desired lending in a region is funneled to another region where the opposite situation prevails.[3]

Modern central banks have become dominant players in the interbank market, which allows them to adjust the sizes of their balance sheets to modify interbank interest rates more directly than in the past. Most central banks set a target rate, or range, for trading in the respective interbank funds markets as the main instrument in the conduct of monetary policy. Although transactions in the interbank market are often unsecured, because of the central banks' guidance the interest rates set in interbank markets, such

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2 Partridge and Rickman (2005) provide evidence about regional business cycles in the U.S., Beck (2021) finds the presence of structural business cycle divergence in Europe, while Poncet and Barthélemy (2008) provides about the different business cycle dynamics across Chinese provinces.

3 Morgan et al. (2004) provide evidence that interstate banking has made U.S. state business cycles smaller, but more alike, while Gong and Kim (2018) find that financial linkages have significant positive effects on the synchronization of regional business cycles. However, significant frictions may hinder the efficient distribution of reserves, such as perceived liquidity or counterparty risk, as experienced in the euro area following the euro area sovereign debt crisis and the following market fragmentation, as discussed in Åberg et al. (2021).

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as the federal funds market in the United States or the interbank market of the Eurosystem, became the benchmark for the short-term interest rates on treasury securities that are considered riskless. Banks hold very large portfolios of short-term securities and exploit arbitrage opportunities arising from the differences among the short-term interest rates of different financial instruments. As a consequence, the divergence between these two interest rates is normally rather limited.[4] Before the financial crisis, in the United States the interest rate on treasuries was normally the lower of the two, and the spread between the fund funds and the treasury interest rates rose during periods of stress in the banking system. The federal funds market and other interbank markets, however, have changed substantially following the financial crisis in 2008, because the Federal Reserve, the ECB, and other major central banks expanded their balance sheet on an unprecedented scale, increasing substantially the amount of reserves in the banking system. To facilitate this process the Federal Reserve began paying interest on these reserve balances, following the lead of the ECB that has paid interest on the deposit facility since the beginning of its operations in 1999. Following these policy innovations and the demand-driven increase in reserves in the context of the financial crisis and the sovereign debt crisis in Europe, the volume of transactions in the fed funds market has decreased, and the interest rate paid on reserve balances has become the primary monetary policy tool.[5]

We model the banking systems of these large monetary areas and the interactions, via the interbank market, among banks, which can operate either locally or in nationwide loan markets, and the central bank. We analyze the deviations from the steady-state general equilibrium values of loan demand and deposits supply produced by exogenous factors, such as productivity shocks. This approach allows us to avoid modeling the real side of the economy, but yields a full financial-sector framework that readily be embedded in a DSGE model. We obtain endogenously all the interest rates of the system and the equilibrium allocations of the banks' assets and liabilities as functions of the exogenous policy choices of the central banks and their subsequent balance sheet allocations. We find that within the context of our banking framework, standard portfolio-separation properties that characterize many models of this class break down, and bank asset and liability allocations become interdependent. In addition, regional loan and deposits markets become interdependent, and regional shocks generate inter-regional spillovers. Interregional transfers via the interbank market can arise when numbers of banks or the relative degrees of market power differ across regions, because banks face different structural costs, wage dynamics, or loan losses, or because loan demand elasticities in the respective markets are different.

To keep the analysis tractable, we assume that risk premia on all kinds of securities are constant. Given that our focus is not on bank risk, we simplify the analysis of the monetary market by assuming a single interbank market, involving both unsecured overnight loan transactions and collateralized repurchase agreement. Relevant divergences between overnight-loan rates and rates on repurchase agreements would in fact emerge only during periods of significant financial stress.[6] An implication is that a central bank's transactions involving repurchase or reverse-repurchase directly provide or subtract liquidity to monetary area's interbank market.[7] Finally, we model interest rates on short-term treasury bills by adding a constant spread to the interbank interest rate. The cornerstone of our model is the interest rate on the interbank market, which is a market price for liquidity that we obtain endogenously and that would exist also in the absence of a central bank, as was the case in the United States during the National Banking era.[8] The central bank of our model, however, has several alternative tools to provide liquidity to the system and to influence open-market and retail bank interest rates throughout the monetary union. We analyze the effects of four different instruments. The first is a change in forward guidance policy that alters the average expected interest rates of the system. The second is a traditional intervention based on open market operations and the associated variations in unremunerated required reserves. The third instrument that we analyze are refinancing operations conducted by the central bank with banking counterparts in the interbank market. The fourth is a change in the interest rate on excess reserves paid by the central bank.

Our model predicts that following the central banks' announcement of higher short-term interest rates that are expected to persist, banks experience higher short-term interest margins and choose to operate with smaller portfolios of loans and deposits, because a smaller size is associated with lower resource costs. However, these responses are nonlinear functions of the banks' market power, hence the response to monetary policy actions is proportionally stronger when the number of banks is larger. In addition, as long as the spread between the interest rate on securities and the interbank interest rate is not influenced by monetary policy actions, banks' holdings of securities remain constant in spite of monetary policy actions. Consequently, deposits holdings respond to monetary policy innovations less than loans.

When conditions differ across regions because of a different number of banks, the regional banks that have more market power operate with a smaller balance sheet, holding a far smaller portfolio of securities than their counterparts, while they still operate on a

[4] See for instance Sarno and Thornton (2003) and Thornton (1994).

[5] See, for instance, Blot et al. (2023).

[6] See Klee et al. (2012). In addition, Afonso et al. (2023) highlights the role played by the Federal Home Loan Banks (FHLBs), which have become the main lenders in the Federal Funds market, because to meet minimum regulatory requirements and to satisfy advances to their members these banks turn to the fed funds market to invest excess cash holdings. However, unlike domestic banks and foreign banks branches, FHLBs do not earn interest on their balances at the central bank, which creates an incentive for them to lend in the federal funds market at rates below the Interest On Reserve Balances (IORB)rate. In turn, this incentive to lend at low rates triggers the arbitrage mechanism between fed funds rates and the IORB rate. In addition, since FHLBs have access to the overnight reverse repo facility that the Fed introduced in 2013, they are unwilling to lend at rates below the overnight reverse repo rate.

[7] Bräuning (2017) provide evidence that the reverse repo facility acts as a buffer for any excess or shortage of liquidity caused by exogenous shocks to total reserves that affect money market interest rates.

[8] The Federal Reserve was established to prevent the banking panics following the strains of the banking system caused by idiosyncratic shocks in the presence of relatively inflexible supplies of money and reserves in the banking system. Carlson and Wheelock (2016) provide evidence that the Fed's establishment substantially reduced seasonal pressures and diminished the importance of interbank connections among national banks, lowering contagion risk in the banking system.

broadly similar scale in the highly remunerative regional retail-lending market. In addition, the smaller-scale banks operating in the region where banks have more market power lend in the interbank market to the banks of the region where banks have less market power, and the latter banks predominate in the less remunerative common retail-lending market. In this environment, interbank lending generates large inter-regional transfers, which generates a higher interbank interest rate that pushes up all short-term interest rates of the system. Restrictive monetary policy actions induce a larger contraction in lending, and monetary policy thereby is more effective in both regions. If banks in one of the two regions experience a cost advantage, they operate with a larger portfolio of securities that is financed by a correspondent reduction in retail lending in the common market. In this case, the higher expected interest rates produced by tighter announced monetary policy produce a stronger contraction in bank lending from the banks that operate at a cost disadvantage, and these banks respond to higher expected interest rates by increasing their holdings of treasury securities.

When central banks use open market operations or repurchase agreements to conduct monetary policy actions, the pattern of results remains broadly similar, but there is one important difference that involves deposits. When future average interest rates are expected to be higher, the amount of unremunerated demand deposits remains unchanged. Remunerated deposits shrink, while the interest rate on these deposits rises. When instead a restrictive monetary policy is conducted by means of open market operations or repurchase agreements, total deposits decline because demand deposits shrink, while the equilibrium amount of remunerated deposits becomes larger because in a higher interest-rates environment deposits become more profitable. A key difference between these two instruments is revealed by our analysis of the dynamics of the system: while the mean reversion of all variables following a shock occurs over a long horizon in both cases, open market operations display far more persistent effects than repurchase-agreement operations, and shocks produced by open market operations have near-permanent effects on banks' balance sheets and interest rates.

When analyzing an abundant reserves environment in which monetary policy is conducted by varying the interest rate on reserves, we find that repurchase-agreement operations are a fundamental tool to mitigate undesired liquidity effects produced by variations in the interest rate on reserves. Higher interest rates on reserves in fact push banks to hold larger deposit balances and in many instances repurchase-agreement operations or reverse repurchase-agreement operations are a necessary complementary tool to reduce the volatility in interbank and securities interest rates. While indeed the two instruments produce effects that go in the same direction on retail lending, they produce opposite effects on interest rates and the amount of deposits.

Our work is closely related to Acharya and Rajan (2022), who develop a banking model in which an interbank market arises because loans are risky, households are risk averse, and banks hold liquidity to face unexpected loan losses that may trigger a run on deposits. They argue that expansion and contractions of the central bank's balance sheet do not necessarily induce symmetric responses in bank deposits and liquidity, because banks may find it optimal to adjust the portfolio of securities to reduce the adjustments of deposits. In our model, the interbank market is influenced by a wider range of factors, including differences in costs or market power across banks, in regional business cycles and in loan demand elasticities. Because we introduce market power in the banking industry and quadratic and adjustment costs on deposits, our banks face stronger incentives to smooth the effects on deposits of any shock. In addition, we model in more detail the interaction between the central bank and the banking industry, and we are able to analyze the role of a large set of policy tools.

Section 2 discusses the relevant literature to which this paper contributes. Section 3 provides an initial exposition of the model that we have developed, initially within the context of a simplified structure that yields tractable analytical expressions and then within a more general setting that includes an interbank market that is analyzed numerically. Section 4 studies monetary policy in a scarce-reserves regime, by analyzing steady-state equilibria and dynamic patterns of the model, while Section 5 analyzes the abundant-reserves environment. Section 6 concludes.

2. Literature

The unprecedented expansions of central banks' balance sheets that followed the financial crisis that began in 2007 and the Covid-induced recession in 2020 have generated strong impacts on banks, and offsetting some of these expansions to fight inflation have brought new challenges. In this section we review the recent literature that has examined the interlinkages among central banks' policy tools and the banking system.

Acharya and Rajan (2022) develop a theoretical model within a three-period competitive-framework in which banks face quadratic costs on risky loans and equity, households are risk averse, and banks desire liquidity to face unexpected loan losses that may trigger runs on deposits. Because the shocks hitting firms and their lenders are idiosyncratic, only some banks are hit by the shocks, and banks trade liquidity in an interbank market in which the risk premium is zero when the economy is healthy, because of diversification. But when financial stress arises, liquidity demands are substantial, and a positive risk premium induces banks with excess liquidity to lend. Acharya and Rajan (2022) argue that when the central bank expands its balance sheet, banks finance the reserves the central bank issues to finance its asset purchases with demand deposits, particularly when reserves are in large supply. In fact, when the central bank buys securities from banks, these banks substitute reserves for securities in their portfolio and the process ends there. But whenever the expansion of the central bank's balance sheet requires it to buy securities from non-banks, banks experience increases in both reserves and deposits, because the non-bank institutions deposit the central bank's newly created funds and initiate the multiplier process. However, when reserves are abundant, a shrinkage in the central bank balance sheet and the corresponding shrinkage in bank-held reserves does not necessarily induce a reduction in deposits. This may occur because banks use part of the reserves to purchase securities or other short-term assets and can thus face a reduction in reserves with a concurrent reduction in the portfolio of short-term assets.

In the context of a model of bank asset allocation, Hogan (2021) argues that when the interest rate on reserves is higher than other short-term rates, banks switch from zero excess reserves to a regime with higher excess reserves and lower lending. Hogan's empirical analysis covering U.S. banks from 2000 through 2018 provides evidence that the switch to a positive excess reserve regime in the post-crisis period accounts for the majority of the decline in bank lending after the financial crisis.

The empirical analysis from Acharya et al. (2022) aims to understand how variations in the size of the Federal Reserve balance sheet affect the deposit base of the banking sector and the associated changes in the size and composition of bank assets. They find that following the FED's balance sheet expansion, banks issued more demand deposits and extended additional loans, while time deposits declined. However, the pattern was not symmetrical following a contraction in the FED's balance sheet, as neither deposits nor credit lines shrank following quantitative tightening policy measures. Somewhat in contrast, and in line with the findings from Diamond et al. (2020) and Hogan (2021) provide evidence that following the large increase in the scale of the Federal Reserve balance sheet, the fraction of illiquid assets held by U.S. banks declined substantially. Diamond et al. (2020) estimate a structural model of the U.S. banking system to analyze the effect of an increase in central bank reserve supply on bank lending, finding that not only a larger amount of reserves does not increase lending but also that reserves crowd out loans.

Stulz et al. (2022) analyzes empirically the determinants of the size of the portfolio of liquid assets of U.S. banks. The authors find that investment motives, particularly the spread between the returns on loans and those on liquid assets, explain the changes in the portfolio, because loans and liquid assets are substitutes. In addition, they find that banks respond to changes in deposits differently depending on their liquid asset holdings and that after 2015, regulatory changes and the payment of interest on reserves become important drivers of the size of this portfolio.

Calomiris et al. (2023) found that in an abundant reserves regime, changes in reserve requirements produce small effects on bank lending. Studying the 1936–37 period, when the Federal Reserve doubled member banks' reserve requirements, they found that higher reserve requirements did not generally increase banks' reserve demand or contract lending because reserve requirements were not binding for most banks.

Changes in reserves operate also by influencing short-term interest rates. Estimating the effects of changes in bank reserves on interest rates, however, is challenging because banks adjusts endogenously excess reserves to respond to interest rate variations.- Bräuning (2017) estimates the liquidity effect produced by shifts in bank reserves by exploiting daily variation in the supply of reserves that are related to changes in the Treasury General Account, which are unrelated to monetary policy or money market conditions and, therefore, qualify as exogenous supply. The estimates provide evidence that a drain in bank reserves generates a significant increase in the effective federal funds spread and the overnight repurchase agreement spread relative to the lower bound of the federal funds target range. Similarly, Smith (2019) found evidence that the liquidity effect still plays a role in an abundant-reserves regime, with exogenous changes in reserves influencing the spread between the federal funds interest rate and the interest on reserves.

More recently, Lopez-Salido and Vissing-Jorgensen (2023) model banks' demand for reserves by assuming that reserves pay interest and produce transaction cost savings as a convenience yield on reserves that is increasing in reserves and decreasing in deposits. In addition, they assume a bank balance sheet cost that is linear in bank assets. An implication of their model, as in ours below, is that the effective federal funds rate equals the interest rate on reserves plus the marginal convenience yield from additional reserves, minus the per-dollar balance sheet cost, because higher balance sheet costs reduce the optimal scale, shifting the reserve demand curve down. The authors also derive the reserve demand relative to repurchase agreements, and they find that the opportunity value of liquidity, measured as the effective federal funds rate minus the interest on excess reserves, is affected not just by the quantity of outstanding reserves but also by the outstanding stock of deposits. In this framework, reserve supply adjusts via private sector take-up decisions to ensure that the market rate clears. They then estimate reserve demand under the assumption that the marginal convenience yield on reserves is linear in log reserves and in log deposits. They find evidence of a stable, very elastic but not flat reserve demand schedule.

This evidence supports the view that variations in the size of central banks balance sheet produce effects on the real side of the economy both via the structure of interest rates and via the effects on the real economy of changes in the size and composition of bank assets. Martin et al. (2013) develop a model to analyze these interactions with a relatively simple bank cost structure and within a perfectly competitive environment. Martin et al. (2013) find that in the absence of frictions, the amount of lending is independent of the amount of reserves in the banking system, whereas in the presence of frictions inducing non-linear costs associated to the size of banks' balance sheets, the effects of large reserves become contractionary. Belongia and Ireland (2024) estimate a structural VAR that separates the Federal Reserve's supply of reserves from the banking system's demand and models the banking system's supply of monetary services. They find that variations in monetary aggregates have effects on economic activity independent of any influence associated with variations in the funds rate alone. Kuang et al. (2024) examine the dynamics of loan growth in response to liquidity changes for banks in the United States with varying levels of reserves, finding that loan growth become more sensitive to changes in banks' overall liquidity levels only when banks are working with ample reserves. Consequently, because the liquidity is not distributed uniformly across the country, counties where banks with ample reserves held a larger market share experienced higher local business growth.

In the euro area, in parallel, to implement monetary policy the ECB used to rely on a corridor system based on two policy rates providing an upper and a lower bound on the marginal lending facility. The ECB's decision to expand its balance sheet with an aim to defeat deflationary pressures and to combat the fragmentation of the money market caused by the great financial crisis and the sovereign debt crisis in the euro area has created a structural excess on bank reserves making the upper bound irrelevant, as discussed by Blot et al. (2023). Consequently, the ECB has introduced a floor system based on the interest rate applied on bank reserves that has been pushed to negative values for several years, while developing several tools to manage its securities portfolio and lend to the banking sector in order to manage the liquidity of the system. As in the United States, the creation of an abundant reserves regime has caused transactions in the interbank market to dry up. However, the interbank market in Europe was facing severe stress because of increased sovereign risk, and to a large extent the decision to adopt quantitative easing policies and expand bank liquidity was dictated by a goal of stabilizing the banking system.[9] In a fragmented system, in fact, the central bank ends up becoming the key player of the system because banks rationed in the money market borrow from the central bank, while banks with excess liquidity lend to the central bank at the remunerated deposit rate, see Åberg et al. (2021).

A key question addressed by several studies with a focus on the euro area regards the optimality of a floor system, like the one established during the quantitative easing period in a context of contractionary monetary policy and increasingly tightened liquidity.[10] Whelan (2023) suggests that a floor system may be more efficient in the new environment than the traditional corridor system, because the larger amount of liquidity in the system has made it more difficult to accurately estimate the demand for bank reserves. The floor system indeed allows the ECB to let the demand side establish the amount of bank reserves, while managing liquidity via refinancing operations with fixed rate full allotment.[11] In addition, Blot et al. (2023) suggest that providing liquidity through full-allotment-refinancing operations allows the ECB to avoid potential distortions on the sovereign debt markets implied by asset purchases. Dabrowski (2023) argues that a return to the previous system, albeit infeasible because it would require a disruptive quantitative tightening, would be desirable because the availability of two separate instruments to manage liquidity and set interest rates may potentially hinder the independence of the central bank. Finally, and more importantly for our work, Baglioni (2024b) argues, on the contrary, that the abundant-reserves approach is desirable also in a context of tighter liquidity, because it gives central banks two independent instruments that can be targeted to different objectives. This policy feature allows the central bank to use the interest rate on reserves as a monetary policy tool, while adjusting the balance sheet to manage liquidity and/or address the potential fragmentation in the context of the euro area. Baglioni's theoretical model suggests that money market rates are insulated from liquidity shocks in a floor system because of the structural excess supply of reserves, and the author provides evidence that deviations from the target level are larger and more volatile when following a corridor rather than a floor system.

Our work is also related to an important empirical literature providing evidence on the regional effects of monetary policy via the banking industry. Segev and Schaffer (2020) found that increased bank competition strengthens the impact of monetary policy on bank loan supply, suggesting that variations in the level of bank competition may have amplified regional asymmetries in the years leading to the Great Recession. Similarly, Buch et al. (2022) find that, after interstate banking deregulation, credit provision rose significantly, while the impact of monetary policy on lending became stronger. Massa and Zhang (2013) provide evidence that the availability of bank financing supply affects firms' responses to monetary policy innovations in different U.S. regions, because the substitutability between bank loans and bonds is limited. More recently Pizzuto (2020) finds that monetary policy tightening in the U.S. leads to a persistent decrease in regional real personal income and employment, with asymmetric effects across regions, but finds weak support for the presence of the credit channels at the regional level. Georgiadis (2015) estimates the transmission of a monetary policy shock across euro area economies, finding that the transmission of monetary policy across euro area economies displays asymmetries driven by differences in economies' structural characteristics. Fielding and Shields (2006) provide evidence of marked heterogeneity in the responses of provincial prices to monetary expansions and contractions in South Africa. Finally, Dia et al. (2023) suggests that the segmentation of the Chinese banking system across provinces reduces the volatility of macroeconomic fluctuations across provinces.

3. The two-region model and its solutions

We contemplate a setting in which banks located in different regional political jurisdictions engage in lending both within local, region-specific markets for retail loans and within a common, cross-region loan market. Without any significant loss of generalization in relation to the issues we address in the present paper, we assume that banks raise deposit funds solely in regional retail markets.

3.1. An initial expositional framework

As a precursor to a more general framework of analysis, we start by laying out, for illustrative purposes only, the simplest possible setup that features both region-specific and common, cross-regional retail loan markets. Within a static setting, let us suppose that banks allocate deposit that they collect within the regions in which they are based between these two alternative classes of loans. The individual bank i in region j, where $j = 1, 2$, is subject to the balance sheet constraint $L^i_\gamma + L^i_{h,\gamma} = D^i_\gamma$, where L^i_γ indicates the amount of loans issued by bank i of region j in the common market, $L^i_{h\gamma}$ is the corresponding amount of lending in the bank's home regional market and D^i_γ is the amount of deposits raised via issuance of regional accounts. Under the assumption that banks within a region have a symmetric cost structure, an individual bank's profit function is given by

$$\pi^i_\gamma = \left[(1-\delta_\gamma) r_L - a_\gamma\right] L^i_\gamma + \left[(1-\delta_{h\gamma}) r_{Lh,\gamma} - a_\gamma\right] L^i_{h,\gamma} - r_{D,\gamma} D^i_\gamma - \left(\frac{\mu_\gamma}{2}\right) \left(L^i\right)^2 - \left(\frac{\mu_{h\gamma}}{2}\right) \left(L^i_{h,\gamma}\right)^2 - \frac{\omega}{2} \left(D^i_\gamma\right)^2, \tag{1}$$

where r_L is the interest rate in the common retail loan market, $r_{Lh,\gamma}$ is the interest rate on the regional loans and $r_{D,\gamma}$ is the interest rate on (regional) deposits. Each dollar of lending within these markets, however, is, subject to respective per-dollar default rates δ_γ

[9] See Blot et al. (2023) and Borio (2023).

[10] See Baglioni (2024a) for a review of this literature.

[11] In line with the operational framework adopted by the Bank of England, see Baglioni (2024a).

and $\delta_{h\gamma}$. The parameter a_γ is the linear component of the bank's cost of nonfinancial resources, such as labor and physical capital, which for simplicity are assumed to depend on lending activities only. The parameters μ_γ and $\mu_{h\gamma}$ are region-specific non-linear resource cost parameters that apply to loans. The parameter ω governs non-linear resource costs applicable to deposits.

For purposes of exposition within this simplified model as well as qualitative and quantitative analysis of the more general framework discussed shortly, we assume that banks engage in Cournot rivalry within the loan and deposit markets in which they operate. Consequently, banks internalize within their optimization choices the loan demand and deposit supply schedules that they confront as well as the responses from their Cournot competitors in those markets. There are n_1 Cournot competitors in the loan and deposits markets of region $j = 1$ and n_2 Cournot rivals in the loan and deposits markets of region $j = 2$. All $n_1 + n_2$ Cournot rivals compete in the common loan market.

We consider the following inverse loan demand and deposit supply schedules of the nonbank public:

$$r_L = -\eta \bar{r}_L \left(\overline{L}\right)^{-1} L^i_\gamma + \xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F + (\eta - \xi + 1)\bar{r}_L, \tag{2}$$

$$r_{Lh,\gamma} = -\eta_{h,\gamma} \bar{r}_{Lh,\gamma} \left(\overline{L}_{h,\gamma}\right)^{-1} L_{h,\gamma} + (\eta + 1)\bar{r}_{Lh,\gamma}, \tag{3}$$

$$r_{D,\gamma} = \varepsilon_\gamma \bar{r}_{D,\gamma} \left(\overline{D}\right)^{-1} D_\gamma + \alpha \bar{r}_{D,\gamma} \left(\bar{r}_F\right)^{-1} r_F - (\varepsilon_\gamma + \alpha - 1)\overline{r_{D,\gamma}}, \tag{4}$$

which, as in Dia and VanHoose (2023) are linear approximations obtained from $L = \left[\Sigma \left(r_L\right)^{-1} \left(r_F\right)^{\xi}\right]^{\frac{1}{\eta}}$, $L_{h,\gamma} = \left[\Sigma_{h,\gamma} \left(r_{Lh,\gamma}\right)^{-1}\right]^{\frac{1}{\eta_\gamma}}$ and $D_\gamma = \left[Z_\gamma r_{D,\gamma} \left(r_F\right)^{-\alpha}\right]^{\frac{1}{\varepsilon_\gamma}}$. The overall amount of lending in the common market is $L = L_1 + L_2$, where the overbar denotes long-run steady-state equilibrium values, η is the inverse elasticity of demand in the common loan market, $\eta_{h,\gamma}$ is the inverse elasticity of demand in the regional loan market, and ε_γ is the inverse elasticity of supply in the regional deposit market. We further assume that borrowers in the common loan market are primarily large firms that can issue commercial paper at rates close to a short run interest rate r_F, which in this initial version of the model could be envisioned as an exogenous rate on money-market securities such as the U.S. T-bills. Hence the loan demand schedule faced by banks contains the cross-elasticity parameter ξ. Similarly, we assume that depositors can access deposit substitutes, such as money market mutual funds, that are remunerated at a rate close to r_F. Hence the deposit supply schedule contains the cross elasticity parameter α.

First order conditions are the following:

$$(1-\delta_\gamma) r_L - a_\gamma - L^i (1-\delta_\gamma)\eta \bar{r}_L \left(\overline{L}\right)^{-1} - \mu_\gamma L^i - \lambda_\gamma = 0. \tag{5}$$

$$(1-\delta_{h\gamma}) r_{Lh,\gamma} - a_\gamma - L^i_{h\gamma} (1-\delta_{h\gamma})\eta \bar{r}_{Lh,\gamma} \left(\overline{L}\right)^{-1} - \mu_{h\gamma} L^i_{h\gamma} - \lambda_\gamma = 0. \tag{6}$$

$$-r_{D,\gamma} - \varepsilon_\gamma \bar{r}_{D,\gamma} \left(\overline{D}\right)^{-1} D^i_\gamma - \omega D^i_\gamma + \lambda^i_\gamma = 0. \tag{7}$$

$$L^i_\gamma + L^i_{h\gamma} - D^i_\gamma = 0. \tag{8}$$

For banks in region 1, the first order conditions can be aggregated *ex post* across the n_1 identical banks as follows[12]:

$$L_{h1} = n_1 L^i_{h1}: \quad \left(n_1+1\right)\left(1-\delta_{h1}\right) r_{Lh,1} - n_1 a_1 - (\eta - \xi + 1)\bar{r}_{Lh1} - \mu_{h1} L_{h1} - n_1 \lambda_1 = 0. \tag{9}$$

$$D_1 = n_1 D^i_1: \quad -n_1 r_{D,1} - n_1 \varepsilon_\gamma \bar{r}_{D,1} \left(\overline{D}\right)^{-1} D^i_1 - n_1 \omega D^i_1 + n_1 \lambda^i_1 = 0. \tag{10}$$

$$\Lambda_1 = n_1 \lambda^i_1: \quad n_1 L^i_1 + n_1 L^i_{1h} - n_1 D^i_1 = 0. \tag{11}$$

After substituting the value of the regional demand for loans and supply of deposits, for, respectively, $n_\gamma L^i_{h\gamma}$, and $n_\gamma D^i_\gamma$, and defining the intercept terms $\overline{R}_{L1} = \left(1-\delta_1\right)(\eta - \xi + 1)\bar{r}_L$, $\overline{R}_{D1} = -(\varepsilon_\gamma + \alpha - 1)\bar{r}_{D1}$, $\overline{R}_{Lh1} = \left(1-\delta_{h1}\right)(\eta+1)\bar{r}_{Lh1}$, these conditions can be rewritten to obtain the aggregate quantities for loans and deposits across each region's banks. The result is a system of eight equations in the six unknown quantities L_1, L_2, L_{h1}, L_{h2}, D_1, D_2 plus the multipliers Λ_1 and Λ_2.

$$\mu_1 L_1 + \Lambda_1 - \left(1-\delta_1\right)\eta \bar{r}_L \left(\overline{L}\right)^{-1} L_2 = \left(n_1+1\right)\left(1-\delta_1\right) r_L - n_1 a_1 - \left(1-\delta_1\right)\xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F - \overline{R}_{L1}, \tag{12}$$

$$\mu_2 L_2 + \Lambda_2 - \left(1-\delta_2\right)\eta \bar{r}_L \left(\overline{L}\right)^{-1} L_1 = \left(n_2+1\right)\left(1-\delta_2\right) r_L - n_2 a_2 - \left(1-\delta_2\right)\xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F - \overline{R}_{L2}, \tag{13}$$

$$\mu_{h1} L_{h1} + \Lambda_1 = (n_1+1)\left(1-\delta_{h1}\right) r_{Lh,1} - n_1 a_1 - \overline{R}_{Lh1}, \tag{14}$$

$$\mu_{h2} L_{h2} + \Lambda_2 = (n_2+1)\left(1-\delta_{h2}\right) r_{Lh,2} - n_2 a_2 - \overline{R}_{Lh2}, \tag{15}$$

[12] See Mathematical Appendix A for the details.

$$-\omega D_1 + \Lambda_1 = (n_1+1) r_{D1} - \alpha \overline{r_D} \left(\bar{r}_F\right)^{-1} r_F + \overline{R}_{D1}, \tag{16}$$

$$-\omega D_2 + \Lambda_2 = (n_1+1) r_{D2} - \alpha \overline{r_D} \left(\bar{r}_F\right)^{-1} r_F + \overline{R}_{D2}, \tag{17}$$

$$L_1 + L_{h1} - D_1 = 0, \tag{18}$$

$$L_2 + L_{h2} - D_2 = 0. \tag{19}$$

In banking models of this form, the presence of non-linear cost terms associated with all relevant assets and liabilities generates portfolio interdependence across the bank's balance sheet, meaning that a bank's choices of assets and liabilities are inherently interrelated and hence must be determined simultaneously.

To illustrate the basic workings of the model, however, we consider the case in which $\omega = 0$, which is a condition sufficient to induce portfolio separation (for a more complete discussion, see, for instance, VanHoose (2022) or the original analysis of this issue provided by Sealey (1985)). The semi-reduced-form solution for the aggregate amount of common-market loans issued from all the n_1 banks in country region $j = 1$ – that is, the solution for given values of the loan and deposit rates, which in general equilibrium also are endogenously determined Cournot-equilibrium values – is given by:

$$L_1 = \frac{K_1 \mu_2 + K_2 \left(1-\delta_1\right) \eta \bar{r}_L \left(\overline{L}\right)^{-1}}{\mu_1 \mu_2 - \left(1-\delta_1\right) \eta \bar{r}_L \left(\overline{L}\right)^{-1} \left(1-\delta_2\right) \eta \bar{r}_L \left(\overline{L}\right)^{-1}}, \tag{20}$$

where K_1 and K_1 are functions of the net interest margins of the banks in the respective regions, as follows:

$$K_1 = \left(n_1+1\right)\left(1-\delta_1\right) r_L - n_1 a_1 - \left(1-\delta_1\right) \xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F - \overline{R}_{L1} - \left(n_1+1\right) r_{D1} + \alpha \overline{r_D} \left(\bar{r}_F\right)^{-1} r_F - \overline{R}_{D1}, \tag{21}$$

and

$$K_2 = \left(n_2+1\right)\left(1-\delta_2\right) r_L - n_2 a_2 - \left(1-\delta_2\right) \xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F - \overline{R}_{L2} - \left(n_1+1\right) r_{D2} + \alpha \overline{r_D} \left(\bar{r}_F\right)^{-1} r_F - \overline{R}_{D2}. \tag{22}$$

Eq. (20) shows that the semi-reduced form response of region 1 banks in the common loan market depends upon all the cost parameters of the banks of both regions. (The response for the aggregate amount of common-market loans issued from all the n_2 banks in region $j = 2$ is symmetrical). Because of the portfolio separation that prevails with $\omega = 0$ asset-allocation decisions are not interrelated, and the responses in the common loan market are independent from those of regional loan markets. Consequently, the only interest rates that influence the banks' decisions about their quantities of common-market lending are the common-market loan rate r_L, the interest rate paid on deposits r_D, and the interest rate on funds obtainable in the open market r_F.

Likewise, the portfolio separation solution for regional lending in region 1 is:

$$L_{h1} = \frac{\left(n_1+1\right)\left(1-\delta_{h1}\right) r_{Lh,1} - n_1 a_1 - \overline{R}_{Lh1} - \left(n_1+1\right) r_{D1} + \alpha \overline{r_D} \left(\bar{r}_F\right)^{-1} r_F - \overline{R}_{D1}}{\mu_{h1}}. \tag{23}$$

The volume of regional lending determined by banks L_{h1} is independent from that for L_1 and this amount depends on the regional loan rate and interest rates on regional deposits and on open-market funds.

The portfolio-separation case described above helps to illustrate how general-equilibrium solutions can be obtained, both in this simple expositional model and in the more general model that we discuss in the next section and analyze in the remainder of the paper. Summing the banks' aggregate desired quantities of common-market lending by banks in both regions, substituting into the nonbank public's demand schedule for common-market loans, and substituting the banks' aggregate desired regional lending amounts into the nonbank public's demand schedules for regional loans yields a system of equations that can be solved for retail-market loan rates in terms of the deposit rates and funds rate. Furthermore, the retail-market deposit rates ultimately can be solved by using (12) through (19) to obtain semi-reduced-form deposit solutions and combining these solutions with the nonbank public's deposit supply schedules. Final reduced-form solutions for loan and deposit quantities can then be obtained from substituting these loan and deposit rate solutions into either the semi-reduced-form solutions for banks' desired quantities or into the nonbank public's loan demand and deposit supply schedules.

In the more general solution with $\omega > 0$, the solutions become interdependent, where magnitudes of the spillovers from one loan market to the other under portfolio interdependence depend on the size of the parameter ω. Naturally, the analytics of obtaining reduced-form solutions becomes more complex in this case, but the essential mechanism for solving the model is the same. Given the complicated setting, we only display the results for L_1 and L_{h1}, which now depend on the positive parameter ω, the other cost parameters, and all relevant rates of return:

$$\begin{aligned} L_1 = {} & [\mu_2(\mu_{h2}+\omega) + \omega\mu_{h1}](\mu_{h1}+\omega)\frac{A_1}{\Delta} + (1-\delta_1)\xi\bar{r}_L \left(\bar{r}_F\right)^{-1}(\mu_{h1}+\omega)(\mu_{h2}+\omega)\frac{A_2}{\Delta} - \omega[\mu_2(\mu_{h2}+\omega)+\omega\mu_{h2}]\frac{A_3}{\Delta} \\ & - \omega^2(1-\delta_1)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}(\mu_{h1}+\omega)(\mu_{h2}+\omega)\frac{A_4}{\Delta} - \mu_{h1}[\mu_2(\mu_{h2}+\omega)+\omega\mu_{h2}]\frac{A_5}{\Delta} - (1-\delta_1)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}\mu_{h2}(\mu_{h1}+\omega)\frac{A_6}{\Delta}, \end{aligned} \tag{24}$$

and that for L_{h1} as

$$L_{h1} = -\omega\mu_2(\mu_{h2}+1)\frac{A_1}{\Delta} - \omega\left[\mu_1\mu_{h2} + 1 + (1-\delta_1)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}(\mu_2+\omega)(\mu_{h2}+\omega)\right]\frac{A_2}{\Delta} + \left(\mu_1\mu_{h2} + \omega\mu_{h2} + \omega^2\mu_{h2} + \omega\mu_1\mu_2\right)\frac{A_3}{\Delta}$$
$$+\,\omega^2\left[\mu_1\mu_2 - (1-\delta_1)(1-\delta_2)\eta^2\overline{r_L^2}\left(\overline{L}\right)^{-2}\right]\frac{A_4}{\Delta} - [\mu_1\mu_2\mu_{h2} + \mu_1\mu_2\omega + \mu_2\mu_{h2}\omega - (\omega+\mu_{h2})(1-\delta_1)(1-\delta_2)\eta^2\overline{r_L^2}\left(\overline{L}\right)^{-2}]\frac{A_5}{\Delta}$$
$$-\,\omega\mu_{h2}(1-\delta_1)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\frac{A_6}{\Delta}, \tag{25}$$

where

$$\Delta = \mu_1[\mu_2(\mu_{h2}+\omega) + \omega\mu_{h2}](\mu_{h1}+\omega) + (1-\delta_1)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\left[(1-\delta_2)\eta\bar{r}_L\left(\overline{L}\right)^{-1}(\mu_{h1}+\omega)(\mu_{h2}+\omega) + \mu_{h1}\mu_{h2}\omega\right]$$
$$+\,\mu_{h2}\mu_{h1}\omega(\mu_{h2}+\omega) + \mu_{h1}\mu_{h2}\omega^2 > 0, \tag{26}$$

and

$$A_1 = (n_1+1)(1-\delta_1)r_L - n_1a_1 - (1-\delta_1)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{L1},$$

$$A_2 = (n_2+1)(1-\delta_2)r_L - (1-\delta_2)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F - n_2a_2 - \overline{R}_{L2},$$

$$A_3 = (n_2+1)(1-\delta_2)r_{Lh1} - n_1a_1 - \overline{R}_{Lh1},$$

$$A_4 = (n_2+1)(1-\delta_2)r_{Lh2} - n_2a_2 - \overline{R}_{Lh2},$$

$$A_5 = (n_1+1)r_{D1} - \alpha\bar{r}_{D,\gamma}\left(\bar{r}_F\right)^{-1}r_F + \overline{R}_{D1},$$

$$A_6 = (n_1+1)r_{D2} - \alpha\bar{r}_{D,\gamma}\left(\bar{r}_F\right)^{-1}r_F + \overline{R}_{D2}.$$

These solutions indicate that both L_1 and L_{h1} are functions of all market interest rates. Hence, common-market lending is now a substitute for lending in regional markets, and regional markets produce spillovers not only in the common market but also in the other regional market. (The solutions for L_2 and L_{h2} are symmetrical). In addition, regional retail deposit rates influence not only lending within the same region's loan market and the common loan market but also within the other regional loan market. All these cross-market interactions are produced by the presence of the positive ω, and it can easily be verified that for $\omega = 0$, Eqs. (24) and (25) simplify to (20) and (23).

3.2. General model

Monetary policy analysis requires a more general framework than the basic model exposited in the previous subsection. Our first step is to broaden banks' balance sheets by including holding of securities $G^i_{\gamma,t}$, which yield a return $r_{G\gamma,t}$ composed of a riskless component equal for all regions that we obtain endogenously. We obtain the equilibrium interbank interest rate endogenously from the offsetting aggregate net positions of banks in different regions F_γ, with $F_1 + F_2 = 0$. We also introduce bank equity funding, denoted E^i_γ, which banks raise by incurring expenses that involve both a linear cost in the form of the required rate of return on equity, denoted r_E, and a non-linear equity-issuance discount component σ. We impose region-specific binding capital requirements θ_γ, which are identical for all classes of loans so that

$$E^i_{1,t} = \theta_1 L^i_{\gamma,t} + \theta_1 L^i_{h1,t}, \tag{27}$$

$$E^i_{2,t} = \theta_2 L^i_{\gamma,t} + \theta_2 L^i_{h2,t}. \tag{28}$$

Banks need to hold unremunerated required reserves $RR^i_{\gamma,t}$ in a fixed proportions to the unremunerated demand deposits $DD^i_{\gamma,t}$, so that

$$RR^i_{1,t} = qDD^i_{1,t}, \tag{29}$$

$$RR^i_{2,t} = qDD^i_{2,t}. \tag{30}$$

Finally, we assume that banks may hold excess reserves $R^i_{\gamma,t}$, which are remunerated by the central bank at the interest rate $r_{R,t}$. In this expanded model, bank i's balance sheet constraint becomes:

$$\left(1-\theta_\gamma\right)L^i_{\gamma,t} + \left(1-\theta_\gamma\right)L^i_{h\gamma,t} + G^i_{\gamma,t} + F^i_{\gamma,t} + R^i_{\gamma,t} = (1-q)DD^i_{\gamma,t} + D^i_{\gamma,t}. \tag{31}$$

Finally, we impose a dynamic structure by assuming that banks are subject to intertemporal costs of adjusting deposits. The bank now maximizes the expected value of a future stream of discounted variable profits:

$$\pi^i_{\gamma,t} = \sum_0^\infty b^t E\left\{\left[\left(1-\delta_{\gamma,t}\right)r_{L,t} - a_{\gamma,t}\right]L^i_{\gamma,t} + \left[\left(1-\delta_{h\gamma,t}\right)r_{Lh,\gamma,t} - a_{\gamma,t}\right]L^i_{h\gamma,t} + r_{F,t}F^i_{\gamma,t} + r_{G,\gamma,t}G^i_{\gamma,t} + r_{R,t}R^i_{\gamma,t} - r_{D,\gamma,t}D^i_{\gamma,t} - r_E E^i_{\gamma,t}\right.$$
$$\left. - \frac{\mu_{\gamma,t}}{2}\left(L^i\right)^2 - \frac{\mu_{h\gamma,t}}{2}\left(L^i_{h\gamma,t}\right)^2 - \frac{\omega}{2}\left(D^i_{\gamma,t}\right)^2 - \frac{\rho}{2}\left(D^i_{j,t} - D^i_{j,t-1}\right)^2 - \frac{\sigma}{2}\left(E^i_{\gamma,t}\right)^2 - \frac{\phi_\gamma}{2}\left(F^i_{\gamma,t}\right)^2 - \frac{\nu_\gamma}{2}\left(G^i_{\gamma,t}\right)^2 - \frac{\tau_\gamma}{2}\left(R^i_{\gamma,t}\right)^2\right\}, \tag{32}$$

where $b^t = \frac{1}{1+r}$ is the discount factor based on a subjective rate of time discount r. The parameter ρ captures deposit-adjustment costs that generate intertemporal dynamics within banks' balance sheets, as in Cosimano (1988), Dia and Giuliodori (2012), and Elyasiani et al. (1995).[13] The parameters σ, τ, ν_γ and ϕ_γ govern the magnitudes of nonlinear resource costs that banks confront in issuing equity, holding excess reserves, managing government securities portfolios, and net positions of interbank borrowings (or lending if negative).

The interest rates on securities are assumed to equal the interbank rate plus region-specific spreads:

$$r_{G1,t} = r_{F,t} + \zeta_1, \tag{33}$$

$$r_{G2,t} = r_{F,t} + \zeta_2, \tag{34}$$

where the spreads $\zeta_1 = \zeta_{11} + \epsilon_t^{\zeta 1}$ and $\zeta_2 = \zeta_{22} + \epsilon_t^{\zeta 2}$ include deterministic components and zero-mean shocks.

As in the simpler model exposited in Section 3.1, we assume that banks are Cournot competitors in retail loan and deposit markets. In loan markets, we specify inverse loan demand functions characterized by market-specific own-elasticity parameters. Cross-elasticity parameters are identical across markets, but we allow loan demand to potentially respond to different interest rates. We assume that borrowers in the national market have access to alternative sources of finance, such as commercial paper in the case of large firms or variable rate mortgages in the case of consumers, whose interest rates track very closely the interbank fund rate. In the case of borrowers in the regional loan market, we assume instead that demand is influenced by the region-specific interest rate on securities $r_{G\gamma}$. Although also these interest rates in most cases follow the interbank interest rate very closely, the assumption permits analysis of the effects of region-specific risk premia and the relative shocks that in the presence of segmented markets may be relevant. Interest rates on sovereign bonds are in fact a benchmark for corporate bonds and, in the example in the European Union the regional markets of our model are sovereign states.[14]

$$r_{L,t} = -\eta \bar{r}_L \left(\overline{L}\right)^{-1} L_t + \xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_{F,t} + (\eta - \xi + 1)\, \bar{r}_L, \tag{35}$$

$$r_{Lh1t} = -\eta_{h1} \bar{r}_{Lh1} \left(\overline{L}\right)^{-1} L_{h1t} + \xi \bar{r}_{Lh1} \left(\bar{r}_{G1}\right)^{-1} r_{G1t} + \left(\eta_{h1} - \xi + 1\right) \bar{r}_{Lh1}, \tag{36}$$

$$r_{Lh2t} = -\eta_{h2} \bar{r}_L h2 \left(\overline{L}\right)^{-1} L_{h2t} + \xi \bar{r}_{Lh2} \left(\bar{r}_{G2}\right)^{-1} r_{G2t} + \left(\eta_{h2} - \xi + 1\right) \bar{r}_{Lh2}. \tag{37}$$

Similarly, the deposit supply schedules depend on region-specific interest rate on securities $r_{G\gamma}$:

$$D_\gamma = {\epsilon_\gamma}^{-1} (\epsilon_\gamma + \alpha - 1) \overline{D} + \overline{D} \left(\epsilon \overline{r_D}\right)^{-1} r_{D,\gamma} - \overline{D} \alpha \left(\epsilon_\gamma \overline{r_{G,\gamma}}\right)^{-1} r_{G,\gamma}, \tag{38}$$

$$r_{D,1} = \epsilon_1 \bar{r}_{D1,t} \left(\overline{D}\right)^{-1} D_{1,t} + \alpha \bar{r}_{D1,t} \left(\overline{r_{G1,t}}\right)^{-1} r_{G1,t} - (\epsilon_1 + \alpha - 1) \bar{r}_{D1,t}, \tag{39}$$

$$r_{D,\gamma} = \epsilon_2 \bar{r}_{D2,t} \left(\overline{D}\right)^{-1} D_{2,t} + \alpha \bar{r}_{D2,t} \left(\overline{r_{G2,t}}\right)^{-1} r_{G2,t} - (\epsilon_2 + \alpha - 1) \bar{r}_{D2,t}. \tag{40}$$

These linearized functions are obtained from the long-run functions $D_\gamma = \left[Z_\gamma r_{D,\gamma} \left(r_F\right)^{-\alpha}\right]^{\frac{1}{\epsilon}}$.

A cornerstone of our model is the inclusion of long-run equilibrium values of loan demand and deposits supply, $\overline{L}$ and $\overline{D}$, respectively, and likewise for the corresponding interest rates. This approach allows avoiding modeling the real side of the economy, but yields a financial-sector framework that could be embedded in a DSGE model. The long-run equilibrium values would arise underlying steady-state equilibria of a DSGE, when productivity or preference shocks have been reabsorbed. Hence, $\overline{L}$ and $\overline{D}$ represent the loan demand and deposit supply associated to the steady-state general equilibrium, while L and D are the corresponding values whenever exogenous factors, such as a productivity shock, generate deviations from the steady-state equilibrium values. Demand deposits $DD_{\gamma,t}$ are unremunerated liabilities, whose quantities, in the time horizon relevant to our analysis, are exogenous (see, for instance Flannery (1982)) and therefore not under the direct control of banks, but rather the outcome of a money multiplier process regulated by the central bank's balance sheet. We initially assume that the central banks creates required reserves in the same proportion in all banks of the two regions, hence $RR_{1,t} = RR_{2,t}$. Although recent work has suggested that the money multiplier may operate with substantial delays, we assume the process of deposit creation to be instantaneous.[15] Because the amount of demand deposits is not under control of individual banks, over the model's horizon, the fixed costs incurred to manage these deposits do not affect portfolio choices, hence our assumption that banks maximize variable profits.

Short-term securities are the main liquid asset in the banks' portfolios, but banks can also sell funds in the interbank market, earning the interest rate $r_{F,t}$, and hold excess reserves when the central bank remunerates excess reserves at the interest rate $r_{R,t}$.

[13] For the empirical relevance of adjustment costs in banking, see also Banerjee et al. (2013).

[14] Massa and Zhang (2013) provides evidence that also in the United States the regional segmentation of the banking industry generates partially insulated local markets, while Huang et al. (2020) finds that in China local public debt crowds out the investment of private firms by tightening their funding constraints.

[15] Ryan and Whelan (2023) analyze the mechanics of the multiplier process. See also the empirical analysis from Carpenter et al. (2015) of the quantitative easing procedures of the FED.

The central bank operates by setting a target for alternative interest rates obtained from a Taylor rule that we do not model. The central bank faces the following balance sheet constraint:

$$F^C + F_{CB} = RR_1 + RR_2 + R_1 + R_2, \tag{41}$$

where F^C is the amount of securities in the asset portfolio and F_{CB} represents the central bank's net repurchase agreements. We assume that the central can dispose of four different monetary instruments:

(a) Forward guidance, conducted by announcing an average desired interest rate $\bar{r}_F$ for an indefinite number of periods: We assume that any change in this target affects in the same proportion all other average expected interest rates. A large literature suggests in fact that changes in wholesale market rates are not fully passed through to short-term bank lending rates, but that instead the pass-through to lending and deposits rates is complete over a three to twelve months horizons.[16]
(b) Open-market operations via the banking system by changing the size its balance sheet and thereby creating required reserves and demand deposits through the money multiplier: We model these interventions under the assumption that the central bank has a target on the short-run interbank interest rate $r_{F,t}$, which is always an endogenous variable in our system.
(c) Innovations in the interest rate $r_{R,t}$ that it sets to remunerate excess reserves.
(d) Repurchase agreement and reverse-repurchase-agreement operations that allow the central bank to temporarily create reserves and to influence the interbank interest rate $r_{F,t}$ without changing the amount of securities held in the portfolio.

In line with the evidence from Dutkowsky and VanHoose (2017), we assume that when the central bank conducts open market operations and varies the size of its balance sheet to achieve a target interest rate in the interbank market, interest rates on excess reserves are below the interbank rate and therefore banks hold zero excess reserves.[17] When instead the central bank sets an interest rate on reserves that is larger than that on interbank funds, no bank is willing to lend in the interbank market, because banks with excess liquidity obtain a better and riskless return from the central bank. However, banks borrow as a counterpart of the repurchase operations conducted by the central bank and because in our model we do not separate the interbank interest rate from the repo rate, the real world counterpart of $r_{F,t}$ in the abundant-reserves regime is the interest rate on repo operations. In this case the central bank uses innovations in the interest rates that remunerates excess reserves as a policy tool, and lets the size of its asset portfolio adjust endogenously.[18] Because the central bank's management of its balance sheet is not the focus of this work, to avoid further complicating the model, we assume that the central bank remunerates excess reserves but not required reserves.[19]

3.2.1. Model solution

In this more general model, we have several sources of interdependence. As before, one of the causes is a non-zero value for the parameter ω, but now a similar role is played also by the non-linear cost-on-funds parameters ϕ_γ. In addition, the non-linear cost of equity σ generates interdependence among common loans, regional loans and security holdings. Finally, via the substitutabilities across the common loan market and the interbank market, the results for any balance sheet item of a bank of a region depend also on all the parameters of the banks of the other region.

Following the procedure illustrated in the former section, after aggregation of the first-order conditions and substitution of the values of the Lagrangian multipliers, we obtain a system of twelve equations in the twelve unknown quantities $L_{1,t}$, $L_{2,t}$, $L_{h1,t}$, $L_{h2,t}$, $G_{1,t}$, $G_{2,t}$, $F_{1,t}$, $F_{2,t}$, $R_{1,t}$, $R_{2,t}$, $D_{1,t}$, $D_{2,t}$:

$$\begin{aligned}&\left(\mu_1+\sigma\theta_1^2\right)L_{1,t}+\sigma\theta_1^2L_{h1,t}-\left(1-\theta_1\right)\phi_1F_{1,t}-\left(1-\delta_{1,t}\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}L_{2,t}=\left(n_1+1\right)\left(1-\delta_{1,t}\right)r_{L,t}-n_1a_{1,t}\\&-\left\{n_1\left(1-\theta_1\right)+\left(1-\delta_{1,t}\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}\right\}r_{F,t}-n_1r_E\theta_1-\left(1-\delta_{1,t}\right)\left(\eta-\xi+1\right)\bar{r}_L.\end{aligned} \tag{42}$$

$$\begin{aligned}&\left(\mu_2+\sigma\theta_2^2\right)L_{2,t}+\sigma\theta_2^2L_{h,2t}-\left(1-\theta_2\right)\phi_2F_{2,t}-\left(1-\delta_{2,t}\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}L_{1,t}=\left(n_2+1\right)\left(1-\delta_{2,t}\right)r_{L,t}-n_2a_{2,t}\\&-\left\{n_2(1-\theta_2)+\left(1-\delta_{2,t}\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}\right\}r_{F,t}-n_2r_E\theta_2-\left(1-\delta_{2,t}\right)\left(\eta-\xi+1\right)\bar{r}_L,\end{aligned} \tag{43}$$

$$\begin{aligned}&\left(\mu_{h1}+\sigma\theta_1^2\right)L_{h1,t}+\sigma\theta_1^2L_{1,t}-(1-\theta_1)\phi F_{1,t}=(n_1+1)\left(1-\delta_{h1}\right)r_{Lh,1t}-(1-\theta_1)n_1r_{F,t}\\&-n_1a_{1,t}-n_1r_E\theta_1-\left(1-\delta_{h1,t}\right)\left[\xi\bar{r}_{Lh1}\left(\bar{r}_{G1}\right)^{-1}r_{G1,t}+\left(\eta_{h1}-\xi+1\right)\bar{r}_{Lh1}\right].\end{aligned} \tag{44}$$

$$\begin{aligned}&\left(\mu_{h2}+\sigma\theta_2^2\right)L_{h2,t}+\sigma\theta_2^2L_{2,t}-(1-\theta_2)\phi F_{2,t}=(n_2+1)\left(1-\delta_{h2}\right)r_{Lh,2t}-\left(1-\theta_2\right)n_2r_{F,t}\\&-n_2a_{2,t}-n_2r_E\theta_2-\left(1-\delta_{h2,t}\right)\left[\xi\bar{r}_{Lh2}\left(\bar{r}_{G2}\right)^{-1}r_{G2,t}+\left(\eta_{h2}-\xi+1\right)\bar{r}_{Lh2}\right].\end{aligned} \tag{45}$$

[16] See, for instance, De Bondt (2005), Gambacorta (2008) or Banerjee et al. (2013).

[17] This is obviously always the case when the central bank pays a zero interest rate on deposits.

[18] Dutkowsky and VanHoose (2017, 2018a, 2018b, 2020) and Fegatelli (2022) explored theoretically the mechanics of these alternative regimes, while Hendrickson (2017) and Hogan (2021) provided supporting empirical evidence. We study the effects of changes in the interest rate on reserves in international loan markets in a companion article.

[19] Similarly, although central banks use repurchase agreements and reverse repurchase-agreements operations that have also non-bank counterparts to change the amount of liquidity in the system without changing the size of the balance sheet, to simplify the analysis we do not model these tools.

$$\phi F_{1,t} + \omega D_{1,t} + \rho(D_{1,t} - D_{1,t-1}) - \rho\beta(D_{1,t+1} - D_{1,t}) = n_1 r_{F,t} - \left(n_1 + 1\right) r_{D1,t} + \alpha \bar{r}_{D1} \left(\bar{r}_{G1}\right)^{-1} r_{G1,t} - (\varepsilon_1 + \alpha - 1)\bar{r}_{D1}, \tag{46}$$

$$\phi_2 F_{2,t} + \omega D_{2,t} + \rho(D_{2,t} - D_{2,t-1}) - \rho\beta(D_{2,t+1} - D_{2,t}) = n_2 r_{F,t} - \left(n_2 + 1\right) r_{D,2} + \alpha \bar{r}_{D2} \left(\bar{r}_{G2}\right)^{-1} r_{G2,t} - (\varepsilon_2 + \alpha - 1)\bar{r}_{D2}, \tag{47}$$

$$G_{1,t} = \frac{\phi_1}{\nu_1} F_{1,t} + \frac{n_1}{\nu_1}(r_{G1,t} - r_{F,t}), \tag{48}$$

$$G_{2,t} = \frac{\phi_2}{\nu_2} F_{2,t} + \frac{n_2}{\nu_2}(r_{G2,t} - r_{F,t}), \tag{49}$$

$$R_1 \geq \frac{n_1 r_R - (n_1 r_F - \phi_1 F_1)}{\left[\tau + \omega\left(1 + RR_1 \left|1 - \frac{1}{q}\right|\right)\right]}. \tag{50}$$

$$R_2 \geq \frac{n_2 r_R - (n_2 r_F - \phi_2 F_2)}{\left[\tau + \omega\left(1 + RR_2 \left|1 - \frac{1}{q}\right|\right)\right]}. \tag{51}$$

$$\left(1 - \theta_1\right) L_{1,t} + \left(1 - \theta_1\right) L_{h1,t} + G_{1,t} + F_{1,t} + R_{1,t} - (1 - q)\, DD_{1,t} - D_{1,t} = 0, \tag{52}$$

$$\left(1 - \theta_2\right) L_{2,t} + \left(1 - \theta_2\right) L_{h2,t} + G_{2,t} + F_{2,t} + R_{2,t} - (1 - q)\, DD_{2,t} - D_{2,t} = 0. \tag{53}$$

These equation have to be solved jointly with Eqs. (35)–(37), (39)–(40) to obtain the five short-term interest rates $r_{L,t}$, $r_{Lh1,t}$, $r_{Lh2,t}$, $r_{D1,t}$, $r_{D2,t}$, plus Eqs. (33) and (34) to solve for the security interest rates $r_{G1,t}$ and $r_{G2,t}$. The interbank interest rate $r_{F,t}$ is obtained by imposing the interbank market equilibrium condition $F^i_{1,t} + F^i_{2,t} + F_{CB,t} = 0$. Finally, the central bank's balance sheet constraint (41), together with the condition $RR_{1,t} = RR_{2,t}$ allow to obtain endogenous values for required reserves $RR_{1,t}$ and $RR_{1,t}$ for an exogenous value of F^C_t. The equilibrium amounts of demand deposits and equity follow from Eqs. (27), (28), (29) and (30).

Hence, the final system is composed of twenty-five equations in twenty-five unknown variables, and it can be solved numerically.[20]

The constraints (50) and (51) can either be equalities or inequalities. They hold as equalities when the interest rate on reserves is larger then that on interbank funds and in this case the amount of excess reserves is positive, and, to close the system, an exogenous value must be imposed on required reserves, while F^C_t becomes an endogenous variable. In the other case, when the interest rate on reserves is smaller then that on interbank funds, required reserves are zero, and the model is closed by imposing an exogenous value of F^C_t.

3.3. Calibration strategy

In our numerical analysis, we use U.S. annual FDIC banking data and monetary data from the Federal Reserve. Given our focus on recent innovations in monetary policy, we calibrate the model to match relatively recent data. We focus on the period beginning in 1987, because from March 1986 interest rate ceilings had been eliminated on all deposits except demand deposits. Because the payment of interest on reserve balances became effective in October 2008, to set the average expected long-run parameter values, we define two alternative sample periods, the 1987–2008 period defining a scarce-reserves regime and the 2009–2022 period of abundant reserves.[21] Although our general version of the model allows the presence of securities markets geographically segmented, we initially focus on the simpler case of a single interest rate for securities in region 1 and 2, in line with the institutional settings of the United States and China (see Table 1).

We normalize the average long-run value of outstanding deposits in each region to 1 and set other balance-sheet quantities as a percentage of deposits, based on the correspondent period average FDIC balance-sheet data. We assume identical capital requirements $\theta_1 = \theta_2 = 0.01$ and that in the long-run regional loans and common-market loans have the same value of 0.5, hence when the long-run value of interbank funds in each country are set to zero because the parameter structures are assumed to be identical in the two regions, the correspondent long-run values of equity are equal to 0.10.

To obtain a figure for the average long-run value of loan rates, we compute the mean of the ratio between the total interest revenue on loans and total loans and we assume this long-run average value to be the same for lending in either the common or the regional markets, so that $\bar{r}_L = \bar{r}_{Lh,1} = \bar{r}_{Lh,2}$.[22] Similarly, we calculate the mean over time of the ratio between the total interest costs on deposits and total deposits, and we assume an identical value for both regional markets $\bar{r}_{D,1} = \bar{r}_{D,2} =$. We set an average long-run interest rate on the interbank market equal to the federal funds interest rate, while we calibrate the average expected interest rate on securities $\bar{r}_{G,1} = \bar{r}_{G,2}$, initially assuming an equal value for both regions, equal to the return on 3-month Treasury bills.

Other details of the calibration follow Dia and VanHoose (2023), and we initially assume identical parameters for banks of both regions. In particular, we calculate the value of net linear resource costs as the difference between the ratio of total non-interest

[20] The derivation of this set of equations is in Mathematical Appendix B.

[21] The calibration of the scarce-reserves regimes can also be performed using longer series and the results are very similar.

[22] The value over the entire 1934–2019 period is 0.0665.

Table 1
Parameter values.

Parameter	Interpretation	Source	Value
r_E	Market required return on bank equity	King (2009)	0.104
σ_γ	Non-linear cost of equity issuance	Hennessy and Whited (2007)	0.004
ω_γ	Non-linear core-deposit cost	Kumar (2018)	0.05
$\mu_\gamma = \mu_{h\gamma}$	Non-linear loan cost	Elyasiani et al. (1995)	0.05
ϕ_γ	Non-linear cost of funds	Assumed	0.005
ν_γ	Non-linear cost of securities	Assumed	0.01
τ_γ	Non-linear cost of reserves	Assumed	0.0001
ρ_γ	Adjustment cost on deposits	Assumed	0.75
$\eta = \eta_\gamma$	Inverse loan demand elasticity	Cosimano and Hakura (2011)	0.8
ϵ	Inverse deposit supply elasticity	Average of recent estimates	1
ξ	Loan demand cross-elasticity	Initial assumption	0
α	Deposit supply cross-elasticity	Initial assumption	0
n_γ	Number of banks	Calibrated	17
θ_γ	Capital requirements	Assumed	0.1

income to total loans and the ratio of total non-interest costs to total loans and we multiply this value by the ratio between total loans and total assets. We obtain the loan-loss parameter by taking the average of the ratio between net loan and lease charge-offs and total loan interest rate income.[23]

We assume $\omega_\gamma = 0.05$ from Kumar (2018) and $\mu_\gamma = \mu_{h\gamma} = 0.05$ from Elyasiani et al. (1995), a required rate of return on equity $r_E = 0.1$, based on findings from King (2009), while the value of the equity cost parameter, $\sigma_d = \sigma_f = 0.04$, is taken from Hennessy and Whited (2007). Changes in the value of the latter parameter generate only marginal variations in overall results relative to our baseline analysis. Because interbank transaction are nowadays largely collateralized, we assume that resource costs on funds are much lower than those on loans and deposits: $\phi_\gamma = 0.005$. Because treasury bonds issued by states or regions that do not have control of monetary policy are risky, we assume a benchmark value $\nu_\gamma = 0.01$, larger than that on interbank funds, but still substantially lower than that on loans and deposits to reflect the lower resource costs required to manage the position. The values of these parameters, however, play a significant role only in the presence of large regional differences in the parameters of the model. Finally, we assume a very small value of $\nu_\gamma = 0.0001$, for the parameter regulating the non-linear costs on reserves.

We choose an inverse loan demand elasticity benchmark value of $\eta = 0.8$, which corresponds to an elasticity value of 1.25, but we experiment with other values.[24] We choose an inverse deposit supply elasticity parameter $\epsilon = 1$, which corresponds to a value of 1 for the direct elasticity, which is a rough average between some recent U.S. estimates.[25] We provide a set of results for the simplest environments assuming a zero value for both the inverse loan demand cross-elasticity ξ, and the deposit-supply cross-elasticity parameter, to illustrate that our results do not depend on specific assumptions about these parameters. Alternative equilibria are possible, however, with positive values of these parameters, and their relevance changes the responses when banks rebalance the portfolio, but the main results do not change and we prefer initially to focus on the simplest possible environment.

We provide results for different number of banks, and we choose the number associated with the best fit with the historical data as a benchmark for further analyses. We assume a value for the adjustment cost parameter $\rho_j = 0.75$ and a very small value of the inverse security supply elasticity, $\kappa_\gamma = 0.03$, giving banks very little market power.

To evaluate the calibration strategy, we assume that the current central bank interest rates on funds is equal to the long-run average so that $r_F = \bar{r}_F$, and we check that the balance-sheet constraint holds in equilibrium. We then verify whether the model-generated values of the net interbank position of the central bank, loan rates, deposit rates, total outstanding loans for both regional and common-market and deposits match the respective long-run values. Our results are robust to substantial changes in any of the calibrated parameters.

4. Scarce-reserves regime

Table 2 displays the information regarding the data values that we use in the numerical analysis to pin-down the long-run expected values of the relevant interest rates. In addition, after normalizing the value of deposits to 1, we define a new variable, total deposits $TD_\gamma = D_\gamma + DD_\gamma$ and we use Federal Reserve data on the ratio between demand and total deposits to pin down the value of demand deposits relative to demand deposits. The ratio of demand to total deposits was equal to 9.2 percent, the amount of demand deposits relative to deposits was 0.1013 in the sample period. The ratio of required reserves to demand deposits, the parameter q of the model, was equal to 0.1685. We use Federal Reserve data for the value of total assets of the central bank, expressed as a ratio to deposits, obtaining $F_C = 0.0155$. This value is equal to the total amount of bank reserves, which given that total reserves are equally split between banks in the two regions, corresponds to a ratio of reserves to deposits of 0.0075 percent.

[23] Using loan-loss provisions instead of net loan and lease charge-offs would provide very similar results.

[24] This value is broadly in line with the average values estimated by Cosimano and Hakura (2011), while values for other countries range from −1.83 for Germany to 5.90 for Sweden.

[25] Tanner et al. (2021) find values for the elasticity parameters between 0 and 0.3, while Egan et al. (2017) find values of 0.56 for insured deposits and 0.16 for uninsured deposits; Dick (2008) or Drechsler et al. (2017) instead find larger values (respectively between 1.8 and 3.0, and 5.3).

Table 2
Average 1987–2008 parameter values.

Parameter	Interpretation	Source	Details	Value
$\bar{r}_L = \bar{r}_{Lh,\gamma}$	Loan rates	FDIC	$\frac{Total\ loan\ interest\ income}{Total\ loans}$	0.0828
$\bar{r}_{D,\gamma}$	Deposit rates	FDIC	$\frac{Total\ deposit\ interest\ cost}{Total\ deposits}$	0.0334
$\bar{r}_F$	Fed funds rates	FRED		0.0474
$\bar{r}_{G,\gamma}$	3 Months Treasury bills rate	FRED		0.0437
a_γ	Linear resource cost	FDIC	$\left(\frac{Total\ non{-}interest\ costs}{Total\ loans} - \frac{Total\ non{-}interest\ income}{Total\ loans}\right)$	0.0201
$\delta_\gamma = \delta_{h\gamma}$	Provisions for loan losses	FDIC	$\frac{Provisions}{Total\ loan\ interest\ income}$	0.1636
DD_γ / TD_γ	Share of demand deposits	FRED	$\frac{Demand\ deposits}{Total\ deposits}$	0.0920
RR_γ / TD_γ	Reserves as a share of deposits	FRED	$\frac{Reserves}{Total\ deposits}$	0.0155
L_γ / TD_γ	Loan to deposits ratio	FDIC	$\frac{Loans}{Total\ deposits}$	0.8456
G_γ / TD_γ	Securities to deposits ratio	FDIC	$\frac{Securities}{Total\ deposits}$	0.2543
q	Money multiplier	FRED	$\frac{Reserves}{Demand\ deposits}$	0.1685
$\zeta_{\gamma\gamma}$	Securities spread		Calibrated	0.0003

We calibrate the parameter $\zeta_{\gamma\gamma}$, the deterministic component of the spread between the rate on securities and the interbank interest rate, to match the equilibrium interest rate to the long-run average. A value $\zeta_{\gamma\gamma} = 0.0001$ provides a perfect fit the interest rate on loans with its long-run average, a value $\zeta_{\gamma\gamma} = 0.0003$ allows to perfectly fit for the interbank interest rate with the corresponding long-run average. Because of this trade-off, we choose an intermediate value of $\zeta_{\gamma\gamma} = 0.0002$ that provides a reasonable compromise.

Table 3 provides numerical results for the equilibrium quantities and interest rates as a function of the different numbers of banks, while all parameters are assumed to be identical in the two regions. The first row displays the long-run average values that represent the numerical targets to be matched. The model fits the long-run average values very well when $n_1 = n_2 = 14$, but the results are very robust as the number of banks changes. The only exception is the interest rate on securities that during this period was below the interest rate on fed funds, while the model assumes the opposite to hold, in line with the evidence for the most recent years. The current version of the model, in fact, does not capture the spikes in federal funds rates produced by time-varying risk factors in periods of stress in the banking industry.

The calibration generates a worse fit when the number of banks is very small, because in this case banks have a strong market power and consequently interest rates on loans, securities and funds become too large, deposit rates too small and equilibrium quantities too small, relative to the long-run benchmark. But even in this case the divergence is moderate, with all balance sheet items remaining meaningful. When instead the number of banks is very high, the market equilibrium amount of securities diverges, because for simplicity we have assumed a constant spread between the interest rate on securities and the interbank interest rate. This divergence disappears when giving banks some degree of market power in the market for securities. Alternatively, the divergence would be avoided if the spread between the two market interest rates declines when the number of banks rises. Given the focus of our work, we avoid these complications to keep the structure as simple as possible.

The banks' market power is an inverse function of their number, hence a larger number of banks is associated with lower interest rates on assets and higher rates on deposits liabilities and with a larger portfolio. Changes in the number of banks do not influence only the size of the portfolio, but also the composition, because banks benefit from different degrees of market power in the different asset classes, with the markets for securities being the more competitive, while the regional lending market the less competitive. When comparing the regional and common lending market, we can observe that as the number of banks decreases, the equilibrium quantity in the regional loan markets declines more and the interest rate in the regional market rises more than the correspondent values in the common market. By contrast, the interest rate in the interbank market does not decline linearly when the number of banks rises, because the net supply of funds in the interbank market depends on the relative degree of market power between the loan and deposit markets.

The equilibrium quantity of securities rises with the number of banks, because when banks have less market power in both the markets for loans and deposits, as is the case here when the number of banks grows, the appeal of securities rises. Consequently, when the equilibrium number of banks is substantial, a larger number of banks becomes associated with smaller equilibrium amounts of loans because securities dominate the portfolios. While the interest rates in the loan markets decline when the number of banks rises, in fact, the interest rate on securities instead follows the interbank rate closely and displays a U-shaped behavior. In addition, when the bank operates on a larger scale, the marginal costs associated with holdings of securities rise far less than those on loans, because we assume the non-linear costs associated with securities to be smaller than those on loans, reflecting their lower risk and the lower resource costs that their management requires.

How do the basic equilibria of the model change when the model's parameters differ across regions? Table 4 provides numerical results for the equilibrium quantities and interest rates for alternative parameter values that differ from the benchmark in region 1 only.

The first row of Table 4 displays the results for the case when the number of banks in region 1, $n_1 = 10$, is smaller than the number of banks in region 2, $n_2 = 14$. In this case, banks operating in region 1 have more market power than their counterparts

Table 3
Numerical results for the equilibrium quantities and interest rates for different numbers of banks, benchmark parameter values.

	L_1	L_2	L_{h1}	L_{h2}	D_1	D_2	G_1	G_2	F_1	F_2	DD_1	DD_2	TD_1	TD_2	r_L	r_{Lh1}	r_{Lh2}	r_{D1}	r_{D2}	r_{G1}	r_{G2}	r_F
Long-run average	**0.47**	**0.47**	**0.47**	**0.47**	**1**	**1**	**0.28**	**0.28**	**0**	**0**	**0.10**	**0.10**	**1.10**	**1.10**	**0.0828**	**0.0828**	**0.0828**	**0.0334**	**0.0334**	**0.0437**	**0.0437**	**0.0474**
$n_1 = n_2 = 1$	0.092 €	0.092 €	0.073 €	0.073 €	0.130 €	0.130 €	0.02	0.02	0	0	0.046	0.046	0.176	0.176	0.1360	0.1387	0.1387	0.0044	0.0044	0.0822	0.0822	0.0820
$n_1 = n_2 = 2$	0.225 €	0.225 €	0.192 €	0.192 €	0.378 €	0.378 €	0.04	0.04	0	0	0.046	0.046	0.424	0.424	0.1170	0.1217	0.1217	0.0126	0.0126	0.0620	0.0620	0.0618
$n_1 = n_2 = 5$	0.365 €	0.365 €	0.337 €	0.337 €	0.693 €	0.693 €	0.1	0.1	0	0	0.046	0.046	0.739	0.739	0.0971	0.1012	0.1012	0.0232	0.0232	0.0483	0.0483	0.0481
$n_1 = n_2 = 10$	0.421 €	0.421 €	0.403 €	0.403 €	0.903 €	0.903 €	0.2	0.2	0	0	0.046	0.046	0.949	0.949	0.0892	0.0918	0.0918	0.0302	0.0302	0.0446	0.0446	0.0444
$n_1 = n_2 = 14$	0.432 €	0.432 €	0.418 €	0.418 €	1.006 €	1.006 €	0.28	0.28	0	0	0.046	0.046	1.052	1.052	0.0877	0.0897	0.0897	0.0336	0.0336	0.0446	0.0446	0.0444
$n_1 = n_2 = 15$	0.432 €	0.432 €	0.419 €	0.419 €	1.028 €	1.028 €	0.3	0.3	0	0	0.046	0.046	1.074	1.074	0.0876	0.0894	0.0894	0.0343	0.0343	0.0447	0.0447	0.0445
$n_1 = n_2 = 20$	0.430 €	0.430 €	0.420 €	0.420 €	1.127 €	1.127 €	0.4	0.4	0	0	0.046	0.046	1.173	1.173	0.0879	0.0893	0.0893	0.0376	0.0376	0.0459	0.0459	0.0457
$n_1 = n_2 = 25$	0.422 €	0.422 €	0.414 €	0.414 €	1.215 €	1.215 €	0.5	0.5	0	0	0.046	0.046	1.261	1.261	0.0890	0.0901	0.0901	0.0406	0.0406	0.0475	0.0475	0.0473

Table 4
Numerical results for the equilibrium quantities and interest rates when parameters differ across regions.

	L_1	L_2	L_{h1}	L_{h2}	D_1	D_2	G_1	G_2	F_1	F_2	DD_1	DD_2	TD_1	TD_2	r_L	r_{Lh1}	r_{Lh2}	r_{D1}	r_{D2}	r_{G1}	r_{G2}	r_F
$n_1 = 10,\ n_2 = 14$	0.348	0.480	0.391	0.405	0.947	1.054	0.240	0.377	0.080	−0.080	0.046	0.046	0.993	1.100	0.0902	0.0935	0.0915	0.0316	0.0352	0.0464	0.0464	0.0462
$a_1 = 0.021,\ a_2 = 0.020$	0.377	0.504	0.422	0.430	0.961	0.961	0.280	0.158	0	0	0.046	0.046	1.007	1.007	0.0864	0.0890	0.0879	0.0321	0.0321	0.0428	0.0428	0.0426
$\delta_1 = \delta_{h1} = 0.1691,\ \delta_2 = \delta_{h2} = 0.1591$	0.385	0.493	0.421	0.428	0.968	0.968	0.280	0.177	0	0	0.046	0.046	1.014	1.014	0.0866	0.0891	0.0882	0.0323	0.0323	0.0431	0.0431	0.0429
$\mu_1 = \mu_{h1} = 0.06,\ \mu_2 = \mu_{h2} = 0.05$	0.415	0.453	0.419	0.421	0.993	0.993	0.280	0.244	0	0	0.046	0.046	1.039	1.039	0.0873	0.0895	0.0891	0.0332	0.0332	0.0440	0.0440	0.0438

Note: When not differently specified, the number of banks is equal in both regions, $n_1 = n_2 = 24$.

from region 2 and therefore they are more profitable. Because of the stronger market power, banks in region 1 charge relatively higher interest rates on regional loans than those of region 2 and pay lower interest rates on deposits. Hence, banks in region 1 choose to operate with a smaller balance sheet that is very profitable also because with a smaller size banks incur lower non-linear resource costs. In addition, banks in region 1 lend to banks in region 2 via the interbank market. The interbank interest rate is higher in comparison to an equilibrium in which the number of banks is equal in both regions, because of the higher demand for interbank funds, and consequently interest rates on securities rise in the same proportion and banks in region 1 can charge a far larger spread between interest rates on loans and deposits than their region-2 counterparts. As a consequence, the optimal asset portfolio for banks of both regions differs substantially from that of the equilibria with an equal number of banks in both regions. Banks in region 1 hold proportionally smaller retail loans in both the more remunerative regional market, and even more in the more competitive common market, they hold a smaller portfolio of securities and lend a substantial amount of resources in the interbank market. Banks in region 2 reduce lending in the regional market and take a larger market share in the common market while holding a very large amount of securities. On their liability side, banks in region 2 not only borrow in the interbank market, but raise larger amounts of deposits than in the equilibria with an identical number of banks, because the higher interbank interest rate spills over to all the other inters rates, making banking intermediation more profitable and therefore in region 2 banks hold proportionally larger balance sheets.

The second row of Table 4 displays the results for the case when linear costs are higher in region 1, while the third row illustrates the results of the case when loan loss provisions are larger in region 1, for both classes of loans. In both cases, the returns from retail lending become lower, and banks in region 1 respond by holding a smaller equilibrium amount of loans in the common market, in comparison to the case in which the costs are identical in both regions, to concentrate lending in the regional market, while banks from region 2 gain market share in the common retail lending market. The securities portfolio that banks in region 1 hold remains identical to that of the case in which the costs are identical in both regions, while banks in region 2 finance the larger retail common lending by shrinking the securities portfolio very substantially. Banks in both regions hold a smaller balance sheet, because interest rates are now lower than in the benchmark case. In contrast with the case of a different number of banks, the portfolio adjustments between the two regions do not occur via the interbank market, but rather by adjustments in the securities portfolios. In addition, given that loan equilibria in aggregate are smaller, the interbank interest rate declines because of the larger available liquidity in the system. A similar equilibrium also arises when regional differences involve the non-linear lending-costs parameters, a case illustrated in the bottom row of the table.

The numerical examples illustrate the behavior of the interbank interest rate in the absence of any intervention from the central bank. The interbank rate is essentially a price for liquidity, and as such it depends on the relative abundance of deposit funds in relation to the demand for loans. Hence, lower costs on deposits, or larger costs on loans put a downward pressure on the interbank interest rate. But because the interbank interest rate provides the opportunity cost for loans, the return on deposit liabilities, and changes one-to-one with the interest rate on securities, its variation induce adjustments in both the composition of the asset portfolio and the size of the liabilities. It follows therefore that any disturbance induces a full readjustment of the portfolio and that in this model all balance sheet quantities and interest rates are fully interdependent and that regional spillovers are ubiquitous.

4.1. The effects of monetary policy

4.1.1. Forward guidance

A large literature suggests that changes in wholesale market rates are not fully passed through to short-term bank lending rates, but that instead the pass-through to lending and deposits rates is complete over a three-to-twelve-month horizon.[26] Hence, we

[26] See, for instance, De Bondt (2005), Gambacorta (2008) or Banerjee et al. (2013).

initially analyze the effects of changes in the average expected interest rate on the interbank market set by the central bank $\bar{r}_F$, under the assumption that these innovations affect in the same proportion all the other average expected interest rates. Table 5 provides numerical results for the equilibrium quantities and interest rates for alternative environments and compares the benchmark results obtained in the former section with those following a 25-basis-points increase in the average expected interbank interest rate that we assume to produce a similar increase in all other interest rates of the system.

The top three rows display the results for the benchmark symmetric case in which $n_1 = n_2 = 14$, while the bottom three rows show the results for an alternative environment in which the number of banks is larger, with $n_1 = n_2 = 20$. In both cases, short-term interest rates rise following the announced monetary tightening, and the equilibrium quantity of both deposits and loans declines. Interest rates on loans rise in both cases proportionally more than the 25 basis points of expected rates, suggesting that market power allows banks to increase their margins following tighter monetary policy. In addition, and somewhat surprisingly, the interest rate on loans rises more when the number of banks is larger, indicating that the effects of monetary policy actions are stronger when banks benefit of lower market power. This result is produced by the range of banks that we have chosen and would be different for very small numbers of banks. This model in fact is different from the benchmark models of the industrial organization of banking literature, because the crucial variable of the system, the interbank interest rate, representing both the return on deposits and the opportunity cost for loans, is endogenous, rather than exogenous as in virtually all of the literature. In our model this rate is a price of liquidity that is influenced by the respective non-linear costs on loans and deposits and all other parameters of the model and it does not decline linearly with the number of banks but it rather present a u-shaped behavior. In the case with $n_1 = n_2 = 14$, because of the stronger market power, the increase in deposits interest rates is smaller then in the case with $n_1 = n_2 = 20$. Deposit interest rates, however, rise less than 25 basis points when the number of banks is small; otherwise, the extent of pass-through is more than unitary. The short-term interest rate in the interbank market and the interest rate on securities rise by 28 and 30 basis points in the respective scenarios. Hence short-term interest rates on both the interbank market and securities rise more than proportionally when higher average interest rates are expected. The overshooting is greater when the number of banks is larger. This result, however, depends on the assumption that bank reserves remain unchanged, because the central bank does not adjust its balance sheet, while normally central banks may use different policy instruments simultaneously. Higher expected average interest rates allow banks to increase short-term interest margins and cause profits to rise, because when interest rates are higher banks exploit their market power to operate with a smaller size that is associated with lower resource costs. The contraction in lending produced by the higher expected interest rates is larger when the number of banks increases and their market power declines, because in our calibration the interbank interest rate is higher when the market is more competitive. In this environment, in addition, monetary policy actions produce effects that change non-linearly with the initial level of the interbank interest rates, while the effects of monetary policy become stronger in higher interest-rates environments, in line with the empirical findings from Borio and Gambacorta (2017). In our model, the non-linear effects of monetary policy actions are produced by the assumption of a constant spread between the interest rate on securities and the interbank interest rate and of a lower non-linear costs for securities, because higher interest rates induce banks to hold more securities and operate with smaller loan portfolios.

Deposits and loans decrease proportionally more as the number of banks increase. The contraction in lending is similar in both the common and regional markets. Following the 25-basis-point increase in average expected interest rates the equilibrium quantities of both classes of loans decline by roughly 70 basis points. The decrease is 80 basis points in the case in which the number of banks is larger. The equilibrium quantity of deposits decreases proportionally less than that of loans, because banks keep portfolios of securities constant in absolute values. The equilibrium amount of securities remains constant, because the spread between the securities and the interbank interest rates does not change. Although this result follows from our modeling assumption, the conclusion is quite general, because while we have strong evidence of market power in the markets for loans and deposits, we are not aware of any evidence that the spread between the interest rates on securities and the interbank interest rate varies with the level of interest rates.

The bottom part of the table illustrates the effects of higher expected interest rates when conditions differ across regions, either because of a different number of banks or because of differences in the linear lending-cost parameters. The first case is characterized by large interbank lending, causing a higher interbank interest rate that pushes up all short-term interest rates. In this environment, restrictive monetary policy actions induce a larger contraction in lending. and monetary policy thereby is more effective. In region 1, in which banks are net lenders in the interbank market, deposits decline less than in region 2, where banks are net borrowers. In the new, higher-interest-rate environment, deposits in region 2 decline more, because banks marginally reduce also the size of the securities portfolio. Hence the contraction in the overall balance sheet is larger. The picture is different when differences between the two regions are produced by higher industrial costs in region 1. In this higher-expected-interest-rates environment, banks in region 2 operate with a larger portfolio of securities that is financed by a correspondent reduction in retail lending, mainly concentrated in the more competitive common market, while region 1 banks gain market share in the common retail-lending market.

4.1.2. Open market operations

Central banks conduct open market operations by purchasing or selling securities and thereby changing the size of their balance sheets and the corresponding amount of bank reserves. In this section, we analyze the effects of these operations by assuming that the central bank reduces the size of its balance sheet by on third, so that now $F_C = 0.0103$ and required reserves decline to 0.0051 in each region. To keep the analysis tractable, we assume that these operations do not directly affect the interest rate on securities, because arbitrages keep the spread between the interest rate on securities and the interbank interest rate constant. Table 6 provides numerical results for the equilibrium quantities and interest rates. The top three rows display the results for the benchmark case in

Table 5

Numerical results for the equilibrium quantities and interest rates, following a 25-basis-point increase in average expected interest rates.

	L_1	L_2	L_{h1}	L_{h2}	D_1	D_2	G_1	G_2	F_1	F_2	DD_1	DD_2	TD_1	TD_2	r_L	r_{Lh1}	r_{Lh2}	r_{D1}	r_{D2}	r_{G1}	r_{G2}	r_F
$n_1 = n_2 = 14$	0.432	0.432	0.418	0.418	1.006	1.006	0.28	0.28	0	0	0.046	0.046	1.052	1.052	0.0877	0.0897	0.0897	0.0336	0.0336	0.0446	0.0446	0.0444
$n_1 = n_2 = 14$ higher rates	0.429	0.429	0.415	0.415	1.001	1.001	0.28	0.28	0	0	0.046	0.046	1.047	1.047	0.0907	0.0928	0.0928	0.0359	0.0359	0.0474	0.0474	0.0472
Change	−0.007	−0.007	−0.007	−0.007	−0.005	−0.005	0.000	0.000	0	0	0.000	0.000	−0.005	−0.005	0.0031	0.0031	0.0031	0.0023	0.0023	0.0028	0.0028	0.0028
$n_1 = n_2 = 20$	0.430	0.430	0.420	0.420	1.127	1.127	0.400	0.400	0	0	0.046	0.046	1.173	1.173	0.0879	0.0893	0.0893	0.0376	0.0376	0.0459	0.0459	0.0457
$n_1 = n_2 = 20$ higher rates	0.427	0.427	0.417	0.417	1.121	1.121	0.400	0.400	0	0	0.046	0.046	1.167	1.167	0.0910	0.0925	0.0925	0.0402	0.0402	0.0488	0.0488	0.0486
Change	−0.008	−0.008	−0.008	−0.008	−0.006	−0.006	0.000	0.000	0	0	0.000	0.000	−0.005	−0.005	0.0032	0.0032	0.0032	0.0026	0.0026	0.0030	0.0030	0.0030
$n_1 = 10,\ n_2 = 14$	0.348	0.480	0.391	0.405	0.947	1.054	0.240	0.377	0.080	−0.080	0.046	0.046	0.993	1.1004	0.0902	0.0935	0.0915	0.0316	0.0352	0.0464	0.0464	0.0462
$n_1 = 10,\ n_2 = 14$, higher rates	0.346	0.477	0.388	0.402	0.942	1.048	0.240	0.376	0.080	−0.080	0.046	0.046	0.988	1.094	0.0933	0.0967	0.0947	0.0338	0.0376	0.0493	0.0493	0.0491
Change	−0.007	−0.007	−0.007	−0.007	−0.005	−0.006	0.000	−0.003	0	0	0.000	0.000	−0.005	−0.006	0.0031	0.0032	0.0032	0.0022	0.0024	0.0029	0.0029	0.0029
$a_1 = 0.021,\ a_2 = 0.020$	0.377	0.504	0.422	0.430	0.961	0.961	0.280	0.158	0	0	0.046	0.046	1.007	1.007	0.0864	0.0890	0.0879	0.0321	0.0321	0.0428	0.0428	0.0426
$a_1 = 0.021,\ a_2 = 0.020$, higher rates	0.375	0.500	0.420	0.427	0.957	0.957	0.280	0.160	0	0	0.046	0.046	1.003	1.003	0.0894	0.0921	0.0910	0.0344	0.0344	0.0456	0.0456	0.0454
Change	−0.004	−0.007	−0.006	−0.006	−0.004	−0.004	0.000	0.013	0	0	0.000	0.000	−0.004	−0.004	0.0030	0.0031	0.0031	0.0023	0.0023	0.0028	0.0028	0.0028

Note: Changes for quantities are expressed as percentages, for interest rates are in differences.

which $n_1 = n_2 = 14$, while the three rows below show the results for an alternative environment in which the number of banks is larger, with $n_1 = n_2 = 20$.

The restrictive monetary policy produces a proportional decline in the equilibrium amount of regional and common-market loans. Total deposits decline proportionally less, because the size of the securities portfolio remains unchanged. The decrease in total deposits is entirely produced by the decline in demand deposits, offsetting a rise in remunerated deposits. This result is in line with the empirical findings from Acharya et al. (2022) that quantitative easing policies produced an expansion in demand deposits and credit lines, matched by a decrease of time deposits balances. In a more competitive environment, monetary policy has a stronger effect on loans but a slightly smaller on total deposits.

The bottom part of the table illustrates the effects of the smaller bank reserves when conditions differ across regions, either because of a different number of banks or because of differences in the linear lending-cost parameters. In the first case, characterized by large interbank lending, in region 1, where banks are net lenders in the interbank market, the effects are different from those of the previous section. Deposits now decline less in region 2, in which banks are net borrowers, because banks in region 2 now hold a marginally larger portfolio of securities. Again, when differences between the two regions are produced by larger real-resource costs within region 1, banks from this region lose market share in the common retail-lending market.

The overall picture emerging is that open market operations produce substantial effects on equilibrium loans and deposits, in spite of very modest changes in equilibrium interest rates. A one-third contraction in reserves in fact produces an increase in the interbank interest rate of just three basis points, and this prediction of the model is not too far from the empirical results from Bräuning (2017) suggesting that a reduction of about 23 percent of the Federal Reserve's balance sheet produces, for a constant overnight reverse repo balance, a 10 basis points increase in the repo spread and a 2 basis points increase in the fed funds spread. However, the contraction is lending predicted by the model is of a similar magnitude of that produced by a 25 basis points increase in expected interest rates that produces even stronger effects on short-term interest rates. Open market operations produce strong effects because they directly affect the size of banks' balance sheets primarily through adjustments in loans and deposits rather than changes in securities portfolios.

4.1.3. Repurchase agreement operations

As an alternative to open market operations, a central bank can engage in repurchase-agreement operations, which provide liquidity to eligible counterparties. In this way, a central bank can keep the interbank interest rate in the desired target range but without altering the size of the portfolio of securities held by the central bank. When the central bank engages in a repurchase agreement (repo) with an eligible counterparty, typically a bank, it agrees to buy securities today with a promise to sell back the securities at a specified price at a specific time in the future, which is the following day in the case of overnight transactions. Symmetrically, a reverse repurchase agreement (reverse repo) in the overnight interbank market is a transaction in which the central bank sells a security to a bank with an agreement to repurchase that same security at a specified price the following day. Hence, reverse repo transactions allow the central bank to drain liquidity without selling securities in its portfolio. The difference between the sale price and the repurchase price implies a rate of interest received by the central bank in the case of a repo operation or paid by the central in the case of a reverse repo transaction. In our model, the increase in the central bank's assets produced by a repo operation is captured by the exogenous variable F_{CB}. Under accounting rules, the bank acting as a counterpart in the repo operation must shield the securities to which the central bank would gain title if the repo transaction is not honored, but otherwise the amount of securities G_γ contemporaneously is unchanged.

Essentially, introducing repo operations in our model requires the net positions of banks and the Fed to sum to zero in the interbank market. Banks' net desired F_γ assets in our model imply an upward-sloping net interbank-funds asset supply curve, to which the central bank's "desired" net asset position, which in its position as a policymaker might be regarded as perfectly inelastic and subject to discretionary shifts, is added, as illustrated in Fig. 1. The equilibrium condition for the funds market that $F_{CB} + F_1 + F_2 = 0$ then means that equilibrium is the point at which the aggregate upward-sloping net interbank-funds asset supply curve crosses the vertical axis, and at that point r_F is determined. An increase in the central bank's "desired" F_{CB} position – that is, a discretionary rightward shift in the perfectly inelastic net position – shifts the upward-sloping net interbank-funds asset supply curve rightward and thereby pushes down the equilibrium r_F along the vertical axis.

In this section we analyze the effects of these operations by assuming that the central bank drains liquidity by conducting repo operations equal to $F_C B = 0.0052$, which generate a one third reduction in total assets, as in the example of the previous section. Table 7 provides numerical results for the equilibrium quantities and interest rates. The top three rows display the results for the benchmark case in which $n_1 = n_2 = 14$, while the three rows below show the results for an alternative environment in which the number of banks is larger, with $n_1 = n_2 = 20$ and the bottom part of the table display the results for the cases in which banks hold non-zero initial net positions within the interbank market.

The results are broadly similar to those obtained in the previous section, hence we focus on the main differences between the two policy instruments. Overall, as compared with a standard open-market operation, a repurchase-agreement operation involving an identical change in the central bank's balance sheet produces generally qualitatively similar but quantitatively larger effects. A key quantitative difference is that while open market operations leave the banks' securities portfolio unchanged, restrictive monetary policy actions conducted by means of repurchase operations push banks to hold larger amounts of securities. Consequently, repurchase operations induce a rebalancing of banks' balance sheets that favors securities over retail lending when monetary policy is restrictive that contributes to the portfolio size effect induced by changes in the size of the central bank's balance sheet and bank reserves.

Table 6

Numerical results for the equilibrium quantities and interest rates, following open market operations that reduce by one third the amount of bank reserves.

	L_1	L_2	L_{h1}	L_{h2}	D_1	D_2	G_1	G_2	F_1	F_2	DD_1	DD_2	TD_1	TD_2	r_L	r_{Lh1}	r_{Lh2}	r_{D1}	r_{D2}	r_{G1}	r_{G2}	r_F
$n_1 = n_2 = 14$	0.432	0.432	0.418	0.418	1.006	1.006	0.28	0.28	0	0	0.046	0.046	1.052	1.052	0.0877	0.0897	0.0897	0.0336	0.0336	0.0446	0.0446	0.0444
$n_1 = n_2 = 14$ higher rates	0.429	0.429	0.415	0.415	1.015	1.015	0.28	0.28	0	0	0.031	0.031	1.045	1.045	0.0880	0.0900	0.0900	0.0339	0.0339	0.0449	0.0449	0.0447
Change	−0.006	−0.006	−0.006	−0.006	0.009	0.009	0.000	0.000	0	0	−0.335	−0.335	−0.006	−0.006	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
$n_1 = n_2 = 20$	0.430	0.430	0.420	0.420	1.127	1.127	0.400	0.400	0	0	0.046	0.046	1.173	1.173	0.0879	0.0893	0.0893	0.0376	0.0376	0.0459	0.0459	0.0457
$n_1 = n_2 = 20$ higher rates	0.428	0.428	0.418	0.418	1.136	1.136	0.400	0.400	0	0	0.031	0.031	1.166	1.166	0.0882	0.0896	0.0896	0.0379	0.0379	0.0462	0.0462	0.0460
Change	−0.005	−0.005	−0.005	−0.005	0.008	0.008	0.000	0.000	0	0	−0.335	−0.335	−0.006	−0.006	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
$n_1 = 10,\ n_2 = 14$	0.348	0.480	0.391	0.405	0.947	1.054	0.240	0.377	0.080	−0.080	0.046	0.046	0.993	1.1004	0.0902	0.0935	0.0915	0.0316	0.0352	0.0464	0.0464	0.0462
$n_1 = 10,\ n_2 = 14$, higher rates	0.346	0.477	0.388	0.402	0.956	1.064	0.240	0.378	0.080	−0.080	0.031	0.031	0.986	1.094	0.0905	0.0938	0.0919	0.0319	0.0355	0.0468	0.0468	0.0466
Change	−0.006	−0.006	−0.006	−0.006	0.009	0.009	0.000	0.004	0	0	−0.335	−0.335	−0.007	−0.006	0.0004	0.0003	0.0004	0.0003	0.0003	0.0004	0.0004	0.0004
$a_1 = 0.021,\ a_2 = 0.020$	0.377	0.504	0.422	0.430	0.961	0.961	0.280	0.158	0	0	0.046	0.046	1.007	1.007	0.0864	0.0890	0.0879	0.0321	0.0321	0.0428	0.0428	0.0426
$a_1 = 0.021,\ a_2 = 0.020$, higher rates	0.374	0.502	0.420	0.427	0.969	0.969	0.280	0.158	0	0	0.031	0.031	1.000	1.000	0.0868	0.0893	0.0882	0.0324	0.0324	0.0431	0.0431	0.0429
Change	−0.006	−0.005	−0.005	−0.005	0.009	0.009	0.000	0.000	0	0	−0.335	−0.335	−0.007	−0.007	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003

Note: Changes for quantities are expressed as percentages, for interest rates are in differences.

Table 7

Numerical results for the equilibrium quantities and interest rates, following repurchase-agreement operations that involve one third of the central bank's assets.

	L_1	L_2	L_{h1}	L_{h2}	D_1	D_2	G_1	G_2	F_1	F_2	DD_1	DD_2	TD_1	TD_2	r_L	r_{Lh1}	r_{Lh2}	r_{D1}	r_{D2}	r_{G1}	r_{G2}	r_F
$n_1 = n_2 = 14$	0.432	0.432	0.418	0.418	1.006	1.006	0.28	0.28	0	0	0.046	0.046	1.052	1.052	0.0877	0.0897	0.0897	0.0336	0.0336	0.0446	0.0446	0.0444
$n_1 = n_2 = 14$ higher rates	0.428	0.428	0.415	0.415	1.017	1.017	0.2813	0.2813	0	0	0.031	0.031	1.048	1.048	0.0881	0.0901	0.0901	0.0340	0.0340	0.0450	0.0450	0.0448
Change	−0.007	−0.007	−0.007	−0.007	0.011	0.011	0.005	0.005	0	0	−0.335	−0.335	−0.004	−0.004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
$n_1 = n_2 = 20$	0.430	0.430	0.420	0.420	1.127	1.127	0.400	0.400	0	0	0.046	0.046	1.173	1.173	0.0879	0.0893	0.0893	0.0376	0.0376	0.0459	0.0459	0.0457
$n_1 = n_2 = 20$ higher rates	0.427	0.427	0.417	0.417	1.138	1.138	0.401	0.401	0	0	0.031	0.031	1.169	1.169	0.0883	0.0897	0.0897	0.0380	0.0380	0.0463	0.0463	0.0461
Change	−0.007	−0.007	−0.007	−0.007	0.010	0.010	0.003	0.003	0	0	−0.335	−0.335	−0.004	−0.004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
$n_1 = 10,\ n_2 = 14$	0.348	0.480	0.391	0.405	0.947	1.054	0.240	0.377	0.080	−0.080	0.046	0.046	0.993	1.1004	0.0902	0.0935	0.0915	0.0316	0.0352	0.0464	0.0464	0.0462
$n_1 = 10,\ n_2 = 14$, higher rates	0.346	0.476	0.388	0.401	0.958	1.067	0.241	0.380	0.083	−0.077	0.031	0.031	0.989	1.097	0.0906	0.0939	0.0920	0.0320	0.0356	0.0469	0.0469	0.0467
Change	−0.008	−0.008	−0.008	−0.008	0.012	0.011	0.005	0.008	0	0	−0.335	−0.335	−0.004	−0.003	0.0005	0.0004	0.0005	0.0004	0.0004	0.0005	0.0005	0.0005
$a_1 = 0.021,\ a_2 = 0.020$	0.377	0.504	0.422	0.430	0.961	0.961	0.280	0.158	0	0	0.046	0.046	1.007	1.007	0.0864	0.0890	0.0879	0.0321	0.0321	0.0428	0.0428	0.0426
$a_1 = 0.021,\ a_2 = 0.020$, higher rates	0.373	0.501	0.419	0.427	0.972	0.972	0.281	0.160	0	0	0.031	0.031	1.002	1.002	0.0869	0.0894	0.0883	0.0325	0.0325	0.0432	0.0432	0.0430
Change	−0.008	−0.006	−0.007	−0.007	0.012	0.012	0.005	0.008	0	0	−0.335	−0.335	−0.004	−0.004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004

Note: Changes for quantities are expressed as percentages, for interest rates are in differences.

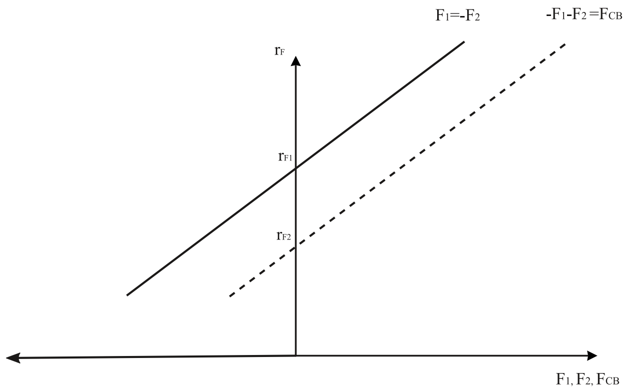

Fig. 1. Equilibrium in the interbank market.

4.1.4. Dynamics

We now analyze the effects of a shock produced either by open market operations changing F_C, or by repurchase-agreement operations changing F_{CB}. To calibrate both shocks, we have used monthly U.S. data spanning from January 1987 to August 2008 for the ratio between total assets of the central bank and total bank deposits. The mean of the series is 0.1685, and the standard deviation is 0.0078. We assume that the shocks follow an AR(1) process, and we impose a value of 0.99 for the AR(1) parameter in our calibration, in line with the parameter estimated using the above U.S. data.

Figs. 2 and 3 show the impulse responses following a one-standard-deviation shock to, respectively, F_C and F_{CB} under the assumption of an identical number of banks and an identical cost structure for banks in the two regions. Because the responses are in this case identical in the two regions, we display the impulse responses for only one of the regions. In both cases the responses are similar. Nevertheless a key difference between the two instruments is the persistence of the shocks. Both shocks are obviously very persistent, given the data used for the calibration, but while F_{CB} shocks are largely reabsorbed in the 50 periods horizon of the impulse response functions, following the F_C shock, all variables are still very far from the zero baseline value after 50 periods.

All the interest rates rise at impact following both shocks and then decline monotonically, with the exception of the interest rates on deposits that follow a hump-shaped pattern, with a peak response occurring very quickly. The responses are stronger following repurchase-agreement innovations.

Deposits similarly display a hump-shaped pattern, with a peak response that is reached very quickly and respond positively to the shocks. However, the negative response of demand deposits is stronger, hence total deposits decline following both shocks. Both classes of retail loans decline at impact and rise monotonically afterwards, by 50 basis points following the F_C shock and by 60 basis points after the F_C shock.

The main difference between the two types of shocks is that following repurchase-agreement operations banks lend to the central bank in the interbank market and increase the amount of securities in the portfolio, albeit by a smaller proportion, while these two balance-sheet items are unaffected by open market operations.

5. Abundant-reserves regime

This section analyzes the different institutional environment established in recent years, after central banks started paying interest on reserves as a policy tool, a period characterized by abundant bank reserves. Table 8 displays the information regarding the data values that we use in the numerical analysis. In this alternative sample all interest rates were far lower than in the previous decades, while loans had a smaller weight in the asset portfolio, offset by a modest increase in the share of securities and a very large increase in the share of reserves. Total reserves were now larger than demand deposits. Consequently, in contrast to the money multiplier process applicable to the previous regime, excess reserves were now remunerated and therefore part of stable equilibrium asset portfolio. To keep the analysis as general as possible, in our baseline model we calibrate the amount of required reserves RR_γ under the assumption that the same money multiplier of the previous period applies, in order to broadly match the ratio of demand deposits to total deposits.[27] As before, we calibrate the parameter $\zeta_{\gamma\gamma}$, the deterministic component of the spread between the rate on securities and the interbank interest rate, to match the equilibrium interest rate to the long-run average and the share of securities in the asset portfolio.

In this regime the central bank has two instruments to reach its policy targets: the interest rate that it pays on excess reserves, r_R, and repurchase-agreement operations. The need for a second instrument arises because the central bank must target the interbank interest rate to avoid that changes in the spread between this last and the interest rate on reserves generate a large volatility in bank assets holdings, and desired reserves in particular. Similarly, the interest rate on securities must remain in close alignment with the

[27] However, when setting required reserves to zero the results change only marginally.

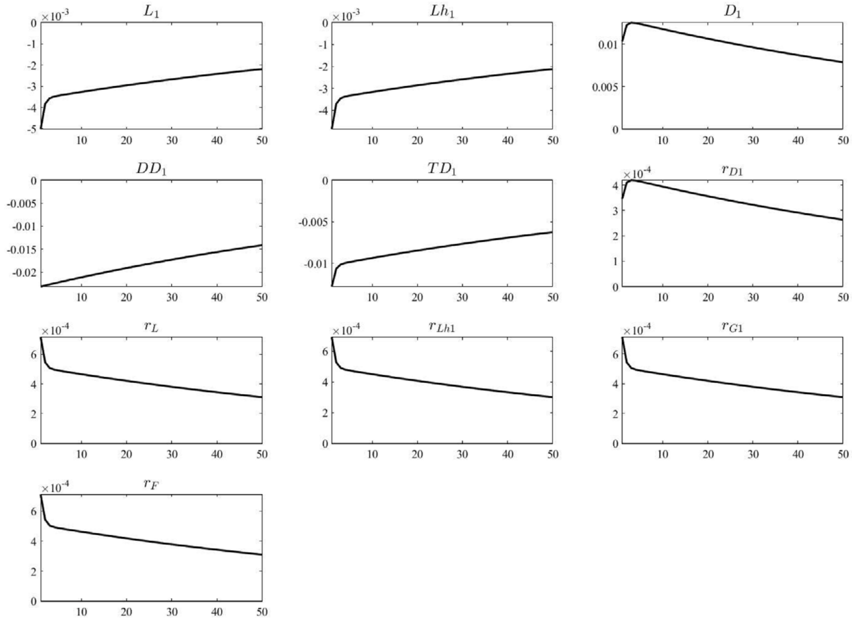

Fig. 2. Impulse responses following a one-standard-deviation negative shock to the central bank's balance sheet via open market operations F_C.

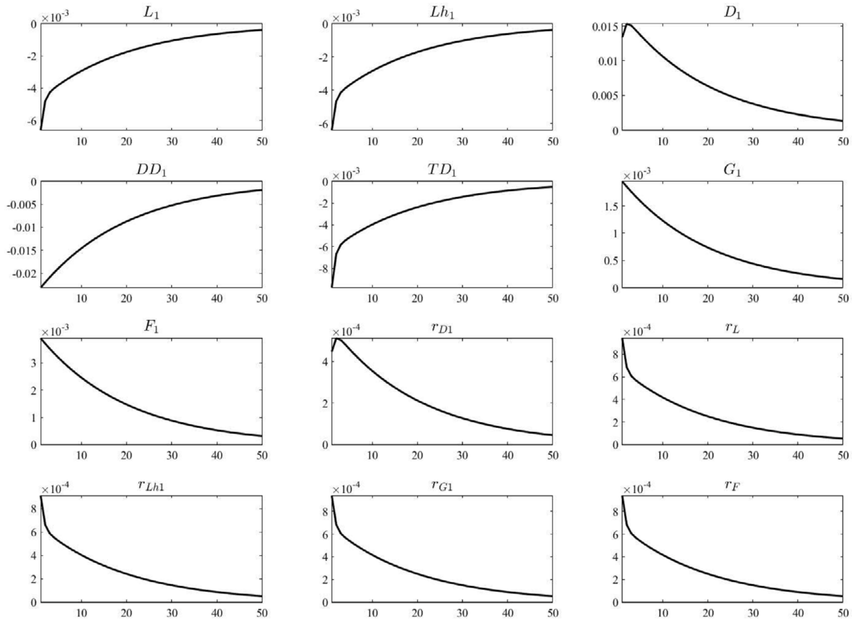

Fig. 3. Impulse responses following a one-standard-deviation negative shock to the central bank's balance sheet via repurchase-agreement operations F_{CB}.

money market rates. The data reported in Table 8 suggest that since the introduction of the new regime these different interest rate have become closely aligned.[28]

Because repos are perfectly substitutable for interbank loans, we regard the interbank loan market as effectively including these transactions. Hence, the implication is that banks essentially exchanges reserves for funds borrowed from the central bank. In the banks' balance sheets $F_\gamma < 0$ and $R_\gamma > 0$ would result, at least temporarily, before any other balance-sheet adjustments that the bank might make. Hence, central bank repurchase agreements temporarily increase the quantity of reserves in the banking system.

28 The table reports only annual averages, but the spreads have remained similarly tight at higher frequencies.

Table 8
Average 2009–2022 parameter values.

Parameter	Interpretation	Source	Details	Value
$\bar{r}_L = \bar{r}_{Lh,\gamma}$	Loan rates	FDIC	$\frac{Total\ loan\ interest\ income}{Total\ loans}$	0.0480
$\bar{r}_{D,\gamma}$	Deposit rates	FDIC	$\frac{Total\ deposit\ interest\ cost}{Total\ deposits}$	0.0044
$\bar{r}_F$	Fed funds rates	FRED		0.0060
$\bar{r}_{G,\gamma}$	3 Months Treasury bills rate	FRED		0.0059
$\bar{r}_R$	Interest rate on excess reserves	FRED		0.0069
a_γ	Net linear resource cost	FDIC	$\left(\frac{Total\ non\text{-}interest\ costs}{Total\ loans} - \frac{Total\ non\text{-}interest\ income}{Total\ loans}\right)$	0.0200
$\delta_\gamma = \delta_{h\gamma}$	Provisions for loan losses	FDIC	$\frac{Provisions}{Total\ loan\ interest\ income}$	0.1591
DD_γ/TD_γ	Share of demand deposits	FRED	$\frac{Demand\ deposits}{Total\ deposits}$	0.1057
RR_γ/TD_γ	Reserves as a share of deposits	FRED	$\frac{Reserves}{Total\ deposits}$	0.1390
L_γ/TD_γ	Loan to deposits ratio	FDIC	$\frac{Loans}{Total\ deposits}$	0.6966
G_γ/TD_γ	Securities to deposits ratio	FDIC	$\frac{Securities}{Total\ deposits}$	0.2719
q	Money multiplier	FRED	$\frac{Reserves}{Demand\ deposits}$	1.3150
$\zeta_{\gamma\gamma}$	Securities spread		Calibrated	0.0001

The main difference between the two instruments is that higher interest rates on reserves require banks to demand more deposits on the liability side, while instead repurchase agreement are financed by the central bank at the interbank interest rate. Hence, *for a given level of reserves*, larger repurchase operations reduce the amount of deposits in the system, because banks substitute deposits with the funds provided in the interbank market by the central bank that are now cheaper. In contrast, therefore, under a reverse repurchase agreement with a bank, the opposite signs will result: $F_C B < 0$, $F_\gamma > 0$, and $R_\gamma < 0$. Hence, central bank reverse repurchase agreements temporarily reduce the quantity of reserves in the banking system.

To define our new initial numerical benchmark we set exogenously an equilibrium number of banks (equal to the one obtained with the previous regime) and we initially assume that the central bank sets an interest rate on reserves that is equal to the long-run average. We then set the quantity of funds provided in the interbank market F_{CB} via repurchase operations to achieve a level of reserves that is line with long-run average. We check the results by assuring first that the balance sheet constraint is respected and then comparing the resulting interbank interest rate with the long-run average and all equilibrium banking quantities and interest rates with the historical averages.

Table 9 provides numerical results for the equilibrium quantities and interest rates when parameters are assumed to be identical for banks in the two regions. The first row displays the long-run average values that represent the numerical targets to be matched. In the second row we display the results assuming that the interest rate on reserves is equal to the long-run average of 69 basis points. In this calibration of the model, this setting requires the central bank to conduct repurchase operations on a very large scale to match the average amount of bank reserves observed in the data. The interest rate solutions match the data reasonably well. In addition, the equilibrium amount of different classes of loans provides a good fit. However the equilibrium amount of deposits is underestimated by twenty-seven percent. The third row provides the results for a slightly different calibration assuming that the central bank sets a higher interest rate on reserves equal to 94 basis points, a value matching the average for the more recent 2014–2022 time span. In this case, the model provides a very good fit for all of the variables, and the amount of repo positions required to match the level of bank reserves observed in the data is a much smaller value of 0.17 of remunerated deposits and roughly twenty percent of total deposits, Furthermore, the results are quite robust as the number of banks changes. The only exception is the interest rate on securities that during the whole period was below the interest rate on fed funds, while the model assumes the opposite to hold. The current version of the model, in fact, does not capture the spikes in federal funds rates produced by time-varying risk factors in periods of stress in the banking industry. Our assumption of larger interest rates on securities than in the interbank market, however, is in line with the evidence for the most recent years. The corresponding total amount of securities in the portfolio of the central bank is of a similar magnitude and matches quite closely the amount of reserves as a share of deposits of both the model and the data.

We will then use the second scenario as a benchmark to analyze the effects of separate changes in monetary policy instruments. The fourth row of Table 9 illustrates the effects of a 25-basis-points increase in the interest rate on excess reserves, while the fifth row shows the changes in comparison to the benchmark. The sixth row illustrates the effect of an increase in repurchase-agreement transaction from 0.17 to 0.34, while the eighth row shows the changes in comparison to the benchmark. The transmission mechanism of monetary policy is different from that of the scarce-reserves regime. A 25-basis-point increase in the interest rate on reserves is now associated with a much smaller, 8-basis-points increase in the interbank interest rates, the government security rate, and both loan rates, while interest rates on deposit rise by 5 basis points only. The extent of interest-rate pass-through is now more limited because a higher rate on reserves induces a large increase in excess reserves and a corresponding increase in deposits. The portfolio of securities remains constant, as long as the spread between the securities rate and the interbank rate is unaffected as

Table 9
Numerical results for the equilibrium quantities and interest rates, following a 25-basis-point increase in interest rates on reserves.

	F_C	F_{CB}	L_1	L_2	L_{h1}	L_{h2}	D_1	D_2	G_1	G_2	F_1	F_2	DD_1	DD_2	TD_1	TD_2	R_1	R_2	r_L	r_{Lh1}	r_{Lh2}	r_{D1}	r_{D2}	r_{G1}	r_{G2}	r_F	r_R
Long-run average	–	–	**0.39**	**0.39**	**0.39**	**0.39**	**1**	**1**	**0.30**	**0.30**	**0**	**0**	**0.12**	**0.12**	**1.12**	**1.12**	**0.15**	**0.15**	**0.0480**	**0.0480**	**0.0480**	**0.0044**	**0.0044**	**0.0059**	**0.0059**	**0.0060**	**0.0069**
Historical data	−0.16	0.45	0.43	0.43	0.42	0.42	0.73	0.73	0.18	0.18	−0.23	−0.23	0.12	0.12	0.85	0.85	0.13	0.13	0.0445	0.0455	0.0455	0.0032	0.0032	0.0057	0.0057	0.0056	0.0069
Recent benchmark	0.18	0.17	0.41	0.41	0.40	0.40	1.00	1.00	0.32	0.32	−0.09	−0.09	0.12	0.12	1.13	1.13	0.15	0.15	0.0465	0.0474	0.0474	0.0044	0.0044	0.0077	0.0077	0.0076	0.0094
Higher r_R	0.44	0.17	0.40	0.40	0.39	0.39	1.12	1.12	0.32	0.32	−0.09	−0.09	0.12	0.12	1.24	1.24	0.29	0.29	0.0473	0.0482	0.0482	0.0049	0.0049	0.0085	0.0085	0.0084	0.0119
Change	0.26	0	−0.02	−0.02	−0.02	−0.02	0.12	0.12	0	0	0	0	0	0	0.10	0.10	0.86	0.86	0.0008	0.0008	0.0008	0.0005	0.0005	0.0008	0.0008	0.0008	0.0025
Higher F_{CB}	0.14	0.22	0.41	0.41	0.40	0.40	0.91	0.91	0.23	0.23	−0.17	−0.17	0.12	0.12	1.03	1.03	0.22	0.22	0.0458	0.0467	0.0467	0.0040	0.0040	0.0070	0.0070	0.0069	0.0094
Change	−0.04	0.29	0.02	0.02	0.02	0.02	−0.09	−0.09	−0.27	−0.27	1.00	1.00	0	0	−0.08	−0.08	0.41	0.41	−0.0007	−0.0007	−0.0007	−0.0004	−0.0004	−0.0007	−0.0007	−0.0007	0

Note: Changes for quantities are expressed as percentages, for interest rates are in differences.

we assumed. The equilibrium amounts of both classes of loans decline by 2 percent. In this regime, the interest rate on reserves does not substitute for the interbank interest rate that remains the opportunity cost for other assets, because banks face an incentive to increase proportionally the amount of deposit liabilities rather than change the compositions of the banks' asset portfolios. A change in the interest rate on reserves influences all other interest rates, but this is a blunt instrument because the spread between the interest rate on reserves and other assets rises substantially when interest rates on reserves rise. For this reason, in this regime the central bank requires a second instrument to offset the undesired liquidity creation associated with higher interest rates on reserves.

Conducting repurchase operations as a complement to changes in the interest rate on reserves allows the central bank to increase bank reserves while generating a reduction in the amount of deposits of the banking system, because banks substitute deposits with the funds provided in the interbank market by the central bank at a lower interest rate. The two instruments for increasing reserves produce opposite effects on interest rates and the amount of deposits. It is important to note, however, that larger repurchase-agreement operations induce a contraction in bank lending, because they produce a large decline in deposits that is matched by a smaller portfolio of all assets different from reserves. The banks' securities portfolios shrink more because they are more sensitive to variations in interest rates, but loan equilibria decline too, albeit to a smaller extent. Hence, both instruments induce a contraction in bank lending, but via very different channels: Higher interest rates on reserves induce banks to substitute reserves for loans in the asset portfolio, while instead repurchase operations induce banks to change the composition of their liabilities and generate a reduction in the scale of the entire asset portfolio.

The availability of two alternative instruments allows the central bank to provide liquidity independently of the target interest rates. When following a new Keynesian monetary policy model, the target interest rates could be set on the basis of a Taylor rule, and the amount of liquidity provided could be adjusted to minimize the volatility of the spreads between different interest rates and the resulting adjustments in bank portfolios. In principle, instead, a monetarist central bank could set a target on liquidity, and use both the interest rate on reserves and repurchase operations to minimize spreads and volatility in bank asset portfolios.

An important caveat is that we do not model other institutions that now play an important role in money markets. In particular, we do not model the choices of money market funds that have access to repurchase and reverse-repurchase operations. The large reverse-repo positions held by the Federal Reserve in recent years have allowed these funds to earn a competitive interest rate on their assets, avoiding a potential run on their liabilities when bank deposit rates where rising. Similarly, we do not model the hedge funds that borrow heavily to arbitrage differences between the interest rates on short-term securities, money market rates and future interest rates on the long-term securities that central banks have bought on a large scale after the financial crisis, because we assume for simplicity that central banks and banks hold short-term securities only with no duration risk.

6. Concluding remarks

In this paper, we have modeled the banking systems of large monetary areas and have obtained solutions for retail loan and deposit rates and for equilibrium allocations of banks' assets and liabilities as functions of the exogenous policy choices of central banks and their subsequent balance sheet allocations. We have analyzed the effects of four alternative monetary policy instruments, focusing in particular on changes in forward guidance policy that alters the expected long-run interest rates, open market and repurchase-agreement operations and associated adjustments in unremunerated required reserves, and changes in the interest rates on excess reserves.

We have determined that when the central bank announces increases in average expected interest rates or operating in a scarce-reserves regime reduces bank reserves via open market or repurchase-agreement operations, banks experience higher short-term interest margins and choose to operate with smaller portfolios, because a smaller size is associated with lower resource costs. However, these responses are a non-linear function of the banks' market power, hence the response to monetary policy actions is proportionally stronger when the initial level of the interbank interest rate is higher. In addition, as long as the spread between the interest rate on securities and the interbank interest is not influenced by monetary policy actions, banks' holdings of securities remain constant in spite of monetary policy actions. Consequently, deposits holdings respond to monetary policy innovations less than loans. When conditions differ across regions because of a different number of banks, the smaller-scale banks operating in the region where banks have more market power lend in the interbank market to the banks of the region where banks have less market power, and the latter banks end up dominating the less remunerative common retail lending market. In this environment, interbank lending generates large inter-regional transfers causing a higher interbank interest rate that, in turn pushes up all short-term interest rates. Restrictive monetary policy actions induce a larger contraction in lending, and thereby monetary policy is more effective in both regions. Whenever instead banks in one of the two regions experience a cost advantage, they operate with a larger portfolio of securities that is financed by a correspondent reduction in retail lending in the common market.

When central banks change future expected average interest rates or the interest rate paid on bank reserves in an abundant-reserves regime, the amount of unremunerated demand deposits remains unchanged, and monetary policy actions influence remunerated deposits that shrink in the first case, or increase in the second, when interbank interest rates rise. When instead a restrictive monetary policy is conducted by means of open market operations or repurchase agreements, the change in total deposits is mainly produced by the variation in demand deposits caused by the operation of the money multiplier. Our analysis of the dynamics of the system reveals that open market operations display far more persistent effects than repurchase-agreement operations, and shocks produced by open market operations have near-permanent effects on banks' balance sheets and interest rates.

In recent years, central banks engineered an abundant-reserves regime in which monetary policy is largely conducted by changing the interest rate on reserves. Our results suggest that in this environment, central banks must use repurchase or reverse-repurchase operations, as a complementary tool. These operations indeed allow a central bank to change the amount of bank reserves and therefore the size of its balance sheet without necessarily inducing a change in the same direction in bank deposits and overall liquidity. When the two instruments are used to change bank reserves in the same direction, they in fact produce opposite effects on interest rates and the amount of deposits. Hence, both instruments induce a similar change in bank lending, but via very different channels: A higher interest rate on reserves induces banks to substitute reserves for loans in the asset portfolio, even if the portfolio becomes larger, while instead repurchase operations induce banks to change the composition of their liabilities and generate a reduction in the scale of the entire assess portfolio.

Our analysis of inter-regional transfers via the banking system is extremely simplified, but the model that we have developed can be used to analyze far more complicated settings, including environments with interest rates differentials produced by different regional spreads on securities, differentials in demand and deposit supply elasticities across regions, or persistent divergences in industrial cost dynamics.

An important caveat is that we have assumed for simplicity that banks hold short-term securities only with no duration risk, and have not modeled explicitly other institutions that now play an important role in money markets, such as money market funds and hedge funds. Similarly, we have not analyzed the effects of the large scale quantitative easing operations that central banks have conducted after the financial crisis by purchasing long-term securities. The absence of maturity mismatches and bank liquidity risks in our model makes the analysis far simpler and more tractable, but this theoretical structure could be extended in future research to introduce these important features.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Mathematical Appendix A

$$\pi_\gamma^i = \left[(1-\delta_\gamma)r_L - a_\gamma\right] L^i + \left[(1-\delta_{h\gamma})r_{Lh,\gamma} - a_\gamma\right] L_{h\gamma}^i - r_{D,\gamma} D_\gamma^i - \frac{\mu_\gamma}{2}\left(L^i\right)^2 - \frac{\mu_{h\gamma}}{2}\left(L_{h\gamma}^i\right)^2 - \frac{\omega}{2}\left(D_\gamma^i\right)^2, \tag{54}$$

s.t. $L_\gamma^i + L_{h\gamma} - D_\gamma = 0$,

and

$$r_L = -\eta \bar{r}_L \left(\overline{L}\right)^{-1} L_\gamma^i + \xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F + (\eta - \xi + 1)\bar{r}_L \tag{55}$$

$$L = -\overline{L}\left(\eta \bar{r}_L\right)^{-1} r_L + \xi \overline{L}\left(\eta \bar{r}_F\right)^{-1} r_F + \eta^{-1}(\eta - \xi + 1)\overline{L} \tag{56}$$

$$r_{D,\gamma} = \varepsilon_\gamma \overline{r_{D,\gamma}}\left(\overline{D}\right)^{-1} D_\gamma + \alpha \overline{r_{D,\gamma}}\left(\bar{r}_F\right)^{-1} r_F - (\varepsilon_\gamma + \alpha - 1)\overline{r_{D,\gamma}} \tag{57}$$

$$D_\gamma = \varepsilon_\gamma^{-1} - (\varepsilon_\gamma + \alpha - 1)\overline{D} + \overline{D}\left(\varepsilon_\gamma \overline{r_D}\right)^{-1} r_{D,\gamma} - \overline{D}\alpha\left(\varepsilon_\gamma \bar{r}_F\right)^{-1} r_F \tag{58}$$

$$r_{Lh,\gamma} = -\eta \bar{r}_{Lh,\gamma}\left(\overline{L}\right)^{-1} L_{h,\gamma} + (\eta + 1)\bar{r}_{Lh,\gamma} \tag{59}$$

$$L_{h,\gamma} = -\overline{L}_{h,\gamma}\left(\eta \bar{r}_{Lh,\gamma}\right)^{-1} r_{Lh,\gamma} + \eta^{-1}(\eta + 1)\overline{L_{h\gamma}} \tag{60}$$

Here $L_{h\gamma}^i$ is the loan quantity in the regional market γ, while L_γ^i is the quantity of the common market. These linearized functions are obtained from $L = \Lambda_\gamma\left[\Sigma\left(r_L\right)^{-1}\left(r_F\right)^{\xi}\right]^{\frac{1}{\eta}}$, $L_{h\gamma} = \left[\Sigma_{h,\gamma}\left(r_{Lh,\gamma}\right)^{-1}\right]^{\frac{1}{\eta}}$ and $D_\gamma = \Omega_\gamma\left[Z_\gamma r_{D,\gamma}\left(r_F\right)^{-\alpha}\right]^{\frac{1}{\varepsilon_\gamma}}$ with $\Lambda_\gamma = \{^{\psi,\gamma=1}_{(1-\psi),\gamma=2}$ and $\Omega_\gamma = \{^{\kappa,\gamma=1}_{(1-\kappa),\gamma=2}$ with $r_{L,\gamma} = \Lambda_\gamma \Sigma_\gamma\left(r_F\right)^{\xi}\left(L_\gamma\right)^{-\eta}$, $r_{Lh,\gamma} = \Sigma_\gamma\left(L_\gamma\right)^{-\eta}$ and $r_{D,\gamma}\left(\Omega_\gamma Z_\gamma\right)^{-1}\left(r_F\right)^{\alpha} D_\gamma{}^{\varepsilon_\gamma}$.

Finally, and $L = L_1 + L_2$ and $\overline{L} = \overline{L_1} + \overline{L_2}$.

The problem becomes the following:

$$\begin{aligned}
\pi_\gamma^i = &\left\{(1-\delta_\gamma)\left[-\eta \bar{r}_L\left(\overline{L}\right)^{-1}\left(L^i + \overset{\wedge}{L^i}\right) + \xi \bar{r}_L\left(\bar{r}_F\right)^{-1} r_F + (\eta - \xi + 1)\bar{r}_L\right] - a_\gamma\right\} L^i \\
&+ \left\{(1-\delta_{h\gamma})\left[-\eta \bar{r}_{Lh,\gamma}\left(\overline{L}\right)^{-1}\left(L_{h\gamma}^i + \overset{\wedge}{L_{h\gamma}^i}\right) + (\eta + 1)\bar{r}_{Lh,\gamma}\right] r_{Lh,\gamma} - a_\gamma\right\} L_{h\gamma}^i \\
&- [\varepsilon_\gamma \bar{r}_{D,\gamma}\left(\overline{D}\right)^{-1}\left(D_\gamma^i + \overset{\wedge}{D_\gamma^i}\right) D_\gamma + \alpha \bar{r}_{D,\gamma}\left(\bar{r}_F\right)^{-1} r_F - (\varepsilon_\gamma + \alpha - 1)\bar{r}_{D,\gamma}] D_\gamma^i \\
&- r_E \theta_\gamma L_\gamma^i - \frac{\mu_\gamma}{2}\left(L^i\right)^2 - \frac{\mu_{h\gamma}}{2}\left(L_{h\gamma}^i\right)^2 - \frac{\omega}{2}\left(D_\gamma^i\right)^2,
\end{aligned} \tag{61}$$

The first order conditions are the following:

$$(1-\delta_\gamma)r_L - a_\gamma - L^i(1-\delta_\gamma)\eta\bar{r}_L\left(\overline{L}\right)^{-1} - \mu_\gamma L^i - \lambda_\gamma = 0. \tag{62}$$

$$(1-\delta_{h\gamma})r_{Lh,\gamma} - a_\gamma - L^i_{h\gamma}(1-\delta_{h\gamma})\eta\bar{r}_{Lh,\gamma}\left(\overline{L}\right)^{-1} - \left(\mu_{h\gamma}\right) L^i_\gamma\lambda_\gamma = 0. \tag{63}$$

$$-r_{D,\gamma} - \epsilon_\gamma\bar{r}_{D,\gamma}\left(\overline{D}\right)^{-1} D^i_\gamma - \omega D^i_\gamma + \lambda^i_\gamma = 0. \tag{64}$$

$$L^i_\gamma + L^i_{h\gamma} - D^i_\gamma = 0. \tag{65}$$

The first order conditions for common loans can be aggregated for the respective number of banks in each market as follows:

$$L_1 = n_1 L^i_1:\ \ n_1\left(1-\delta_1\right)r_L - n_1 a_1 - n_1 L^i_1\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} - n_1\mu_1 L^i_1 - n_1\lambda_1 = 0. \tag{66}$$

$$L_2 = n_2 L^i_2:\ \ n_2\left(1-\delta_2\right)r_L - n_2 a_2 - n_2 L^i_2\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} - n_2\mu_2 L_2{}^i - n_2\lambda_2 = 0. \tag{67}$$

And given that $r_L = -\eta\bar{r}_L\left(\overline{L}\right)^{-1} L + \xi\bar{r}_L\left(\bar{r}_F\right)^{-1} r_F + (\eta-\xi+1)\bar{r}_L$, it follows that

$$r_L = -\left(L_1+L_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} + \xi\bar{r}_L\left(\bar{r}_F\right)^{-1} r_F + (\eta-\xi+1)\bar{r}_L \tag{68}$$

and

$$-L_1\eta\bar{r}_L\left(\overline{L}\right)^{-1} = r_L + L_2\eta\bar{r}_L\left(\overline{L}\right)^{-1} - \xi\bar{r}_L\left(\bar{r}_F\right)^{-1} r_F - (\eta-\xi+1)\bar{r}_L \tag{69}$$

Hence since $L_1 = n_1 L^i_1$ and $L_2 = n_2 L^i_2$:

$$\begin{aligned}&n_1\left(1-\delta_1\right)r_L - n_1 a_1\\&+\left(1-\delta_1\right)\left[r_L + L_2\eta\bar{r}_L\left(\overline{L}\right)^{-1} - \xi\bar{r}_L\left(\bar{r}_F\right)^{-1} r_F - (\eta-\xi+1)\bar{r}_L\right]\\&-n_1\left(\mu_1+\beta_1 c_1+\sigma\theta_1^2\right)L^i_1 - n_1\lambda_1 = 0.\end{aligned} \tag{70}$$

$$\begin{aligned}&\left(1-\delta_2\right)r_L - n_2 a_2\\&+\left(1-\delta_2\right)\left[r_L + L_1\eta\bar{r}_L\left(\overline{L}\right)^{-1} - \xi\bar{r}_L\left(\bar{r}_F\right)^{-1} r_F - (\eta-\xi+1)\bar{r}_L\right]\\&-n_2\mu_2 L_2{}^i - n_2\lambda_2 = 0.\end{aligned} \tag{71}$$

And,

$$\begin{aligned}&L_1 = n_1 L^i_1:\ \ \left(n_1+1\right)\left(1-\delta_1\right)r_L - n_1 a_1 + \left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} L_2\\&-\left(1-\delta_1\right)\left[\xi\bar{r}_L\left(\bar{r}_F\right)^{-1} r_F + (\eta-\xi+1)\bar{r}_L\right] - \mu_1 L_1 - n_1\lambda_1 = 0.\end{aligned} \tag{72}$$

$$\begin{aligned}&L_2 = n_2 L^i_2:\ \ \left(n_2+1\right)\left(1-\delta_2\right)r_L - n_2 a_2 + \left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} L_1\\&-\left(1-\delta_2\right)\left[\xi\bar{r}_L\left(\bar{r}_F\right)^{-1} r_F + (\eta-\xi+1)\bar{r}_L\right] - \mu_2 L_2 - n_2\lambda_2 = 0.\end{aligned} \tag{73}$$

The other first order conditions can be aggregated as follows:

$$L_{h1} = n_1 L^i_{h1}:\ \ \left(n_1+1\right)\left(1-\delta_{h1}\right)r_{Lh,1} - n_1 a_1 - \left(\eta_{h1}-\xi+1\right)\bar{r}_{Lh1} - \mu_{h1}L_{h1} - n_1\lambda_1 = 0. \tag{74}$$

$$D_1 = n_1 D^i_1:\ \ -n_1 r_{D,1} - n_1\epsilon_{h1}\bar{r}_{D,1}\left(\overline{D}_1\right)^{-1} D^i_1 - n_1\omega D^i_1 + n_1\lambda^i_1 = 0. \tag{75}$$

$$\Lambda_1 = n_1\lambda^i_1:\ \ n_1 L^i_1 + n_1 L^i_{1h} - n_1 D^i_1 = 0. \tag{76}$$

After substituting the value of the demand for loans and the supply of deposits, for, respectively, $n_\gamma L^i_{h\gamma}$, and $n_\gamma D^i_\gamma$, these conditions can be rewritten to obtain the aggregate quantities for each region as follows:

$$\begin{aligned}&L_{h1} = n_1 L^i_{h1}:\ \ (n_1+1)\left(1-\delta_{h1}\right)r_{Lh,1} - n_1 a_1\\&-\left(1-\delta_{h1}\right)\left(\eta_{h1}+1\right)\bar{r}_{Lh,1} - \mu_{h1}L_{h1} - n_1\lambda_1 = 0.\end{aligned} \tag{77}$$

$$-n_1 r_{D,1} - \epsilon_{h,1}\bar{r}_{D,1}\left(\overline{D}_1\right)^{-1}\left[\epsilon^{-1}_{h,1}(\epsilon_{h,1}+\alpha-1)\overline{D}_1 + \overline{D}_1\left(\epsilon_{h,1}\bar{r}_{D,1}\right)^{-1} r_{D,1} - \overline{D}_1\alpha\left(\epsilon_{h,1}\bar{r}_F\right)^{-1} r_F\right] - n_1\omega D^i_1 + n_1\lambda^i_1 = 0. \tag{78}$$

And finally, after defining the intercept terms

$$\overline{R}_{L1} = \left(1-\delta_1\right)(\eta-\xi+1)\bar{r}_L$$

$$\overline{R}_{D1} = (\varepsilon_{h,1}+\alpha-1)\bar{r}_{D1},$$

$$\overline{R}_{Lh1} = \left(1-\delta_{h1}\right)\left(\eta_{h1}+1\right)\bar{r}_{Lh1}$$

we obtain:

$$\mu_1 L_1 + \Lambda_1 - \left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}L_2 = \left(n_1+1\right)\left(1-\delta_1\right)r_L - n_1 a_1 - \left(1-\delta_1\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{L1}. \tag{79}$$

$$\mu_2 L_2 + \Lambda_2 - \left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}L_1 = \left(n_2+1\right)\left(1-\delta_2\right)r_L - n_2 a_2 - \left(1-\delta_2\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{L2}. \tag{80}$$

$$\mu_{h1}L_{h1} + \Lambda_1 = (n_1+1)\left(1-\delta_{h1}\right)r_{Lh,1} - n_1 a_1 - \overline{R}_{Lh1}. \tag{81}$$

$$\mu_{h2}L_{h2} + \Lambda_2 = (n_2+1)\left(1-\delta_{h2}\right)r_{Lh,2} - n_2 a_2 - \overline{R}_{Lh2}. \tag{82}$$

And similarly,

$$-\omega D_1 + \Lambda_1 = \left(n_1+1\right)r_{D1} - \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F + \overline{R^1_D}. \tag{83}$$

$$-\omega D_2 + \Lambda_2 = \left(n_1+1\right)r_{D2} - \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F + \overline{R}_{D2}. \tag{84}$$

Which together with
and

$$L_1 + L_{h1} - D_1 = 0, \tag{85}$$

$$L_2 + L_{h2} - D_2 = 0, \tag{86}$$

form a system of eight equations in the six unknown quantities L_1, L_2 L_{h1}, L_{h2}, D_1, D_2 plus the multipliers Λ_1 and Λ_2. The system can be reduced to six equations in six unknowns by substituting the value of the multipliers: $\Lambda_1 = \omega D_1 + \left(n_1+1\right)r_{D1} - \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F + \overline{R^1_D}$. and $\Lambda_2 = \omega D_2 + \left(n_1+1\right)r_{D2} - \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F + \overline{R}_{D2}$.

$$\begin{aligned}
&\mu_1 L_1 - \left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}L_2 + \omega D_1 = \left(n_1+1\right)\left(1-\delta_1\right)r_L - n_1 a_1 \\
&-\left(1-\delta_1\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{L1} - \left(n_1+1\right)r_{D1} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R^1_D}. \\
&\mu_2 L_2 - \left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}L_1 + \omega D_2 = \left(n_2+1\right)\left(1-\delta_2\right)r_L - n_2 a_2 \\
&-\left(1-\delta_2\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{L2} - \left(n_1+1\right)r_{D2} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{D2}.
\end{aligned} \tag{87}$$

$$\mu_{h1}L_{h1} + \omega D_1 = (n_1+1)\left(1-\delta_{h1}\right)r_{Lh,1} - n_1 a_1 - n_1\tau_1 - \overline{R}_{Lh1} - \left(n_1+1\right)r_{D1} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R^1_D}. \tag{88}$$

$$\mu_{h2}L_{h2} + \omega D_2 = (n_2+1)\left(1-\delta_{h2}\right)r_{Lh,2} - n_2 a_2 - n_2\tau_2 - \overline{R}_{Lh2} - \left(n_1+1\right)r_{D2} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{D2}. \tag{89}$$

$$L_1 + L_{h1} - D_1 = 0, \tag{90}$$

$$L_2 + L_{h2} - D_2 = 0, \tag{91}$$

The system is

$$\begin{aligned}
&\begin{pmatrix}
\mu_1 & -\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & 0 & 0 & \omega & 0 \\
-\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & \mu_2 & 0 & 0 & 0 & \omega \\
0 & 0 & \mu_{h1} & 0 & \omega & 0 \\
0 & 0 & 0 & \mu_{h2} & 0 & \omega \\
1 & 0 & 1 & 0 & -1 & 0 \\
0 & 1 & 0 & 1 & 0 & -1
\end{pmatrix}
\begin{pmatrix} L_1 \\ L_2 \\ L_{h1} \\ L_{h2} \\ D_1 \\ D_2 \end{pmatrix} \\
&= \begin{pmatrix}
\left(n_1+1\right)\left(1-\delta_1\right)r_L - n_1 a_1 - \left(1-\delta_1\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{L1} - \left(n_1+1\right)r_{D1} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R^1_D} \\
\left(n_2+1\right)\left(1-\delta_2\right)r_L - n_2 a_2 - \left(1-\delta_2\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{L2} - \left(n_1+1\right)r_{D2} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{D2} \\
(n_1+1)\left(1-\delta_{h1}\right)r_{Lh,1} - n_1 a_1 - \overline{R}_{Lh1} - \left(n_1+1\right)r_{D1} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R^1_D} \\
(n_2+1)\left(1-\delta_{h2}\right)r_{Lh,2} - n_2 a_2 - \overline{R}_{Lh2} - \left(n_1+1\right)r_{D2} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{D2} \\
0 \\
0
\end{pmatrix}
\end{aligned} \tag{92}$$

The version with $\omega = 0$ becomes

$$\begin{pmatrix} \mu_1 & -\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & 0 & 0 & 0 & 0 \\ -\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & \mu_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_{h1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_{h2} & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} L_1 \\ L_2 \\ L_{h1} \\ L_{h2} \\ D_1 \\ D_2 \end{pmatrix}$$
$$= \begin{pmatrix} \left(n_1+1\right)\left(1-\delta_1\right)r_L - n_1 a_1 - \left(1-\delta_1\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{L1} - \left(n_1+1\right)r_{D1} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R^1_D} \\ \left(n_2+1\right)\left(1-\delta_2\right)r_L - n_2 a_2 - \left(1-\delta_2\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{L2} - \left(n_1+1\right)r_{D2} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{D2} \\ \left(n_1+1\right)\left(1-\delta_{h1}\right)r_{Lh,1} - n_1 a_1 - \overline{R}_{Lh1} - \left(n_1+1\right)r_{D1} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R^1_D} \\ \left(n_2+1\right)\left(1-\delta_{h2}\right)r_{Lh,2} - n_2 a_2 - \overline{R}_{Lh2} - \left(n_1+1\right)r_{D2} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{D2} \\ 0 \\ 0 \end{pmatrix} \tag{93}$$

The determinant is

$$\mu_{h1}\begin{vmatrix} \mu_1 & -\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & 0 & 0 & 0 \\ -\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & \mu_2 & 0 & 0 & 0 \\ 0 & 0 & \mu_{h2} & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & -1 \end{vmatrix} =$$

$$-\mu_{h1}\begin{vmatrix} \mu_1 & -\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & 0 & 0 \\ -\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & \mu_2 & 0 & 0 \\ 0 & 0 & \mu_{h2} & 0 \\ 1 & 0 & 0 & -1 \end{vmatrix} =$$

$$\mu_{h1}\begin{vmatrix} \mu_1 & -\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & 0 \\ -\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & \mu_2 & 0 \\ 0 & 0 & \mu_{h2} \end{vmatrix} =$$

$$\mu_{h1}\mu_{h2}\begin{vmatrix} \mu_1 & -\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} \\ -\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & \mu_2 \end{vmatrix} =$$

$$\mu_{h1}\mu_{h2}\left\{\mu_1\mu_2 - \left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\right\}$$

Defining

$$\begin{pmatrix} \left(n_1+1\right)\left(1-\delta_1\right)r_L - n_1 a_1 - \left(1-\delta_1\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{L1} - \left(n_1+1\right)r_{D1} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R^1_D} \\ \left(n_2+1\right)\left(1-\delta_2\right)r_L - n_2 a_2 - \left(1-\delta_2\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{L2} - \left(n_1+1\right)r_{D2} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{D2} \\ \left(n_1+1\right)\left(1-\delta_{h1}\right)r_{Lh,1} - n_1 a_1 - \overline{R}_{Lh1} - \left(n_1+1\right)r_{D1} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R^1_D} \\ \left(n_2+1\right)\left(1-\delta_{h2}\right)r_{Lh,2} - n_2 a_2 - \overline{R}_{Lh2} - \left(n_1+1\right)r_{D2} + \alpha\overline{r_D}\left(\bar{r}_F\right)^{-1}r_F - \overline{R}_{D2} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} K_1 \\ K_2 \\ H_1 \\ H_2 \\ 0 \\ 0 \end{pmatrix}$$

To get the numerator of the solution for L_1, we need to calculate the following determinant:

$$\begin{vmatrix} K_1 & -\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & 0 & 0 & 0 & 0 \\ K_2 & \mu_2 & 0 & 0 & 0 & 0 \\ H_1 & 0 & \mu_{h1} & 0 & 0 & 0 \\ H_2 & 0 & 0 & \mu_{h2} & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \end{vmatrix} =$$

$$\begin{vmatrix} K_1 & -\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & 0 & 0 \\ K_2 & \mu_2 & 0 & 0 \\ H_1 & 0 & \mu_{h1} & 0 \\ H_2 & 0 & 0 & \mu_{h2} \end{vmatrix} =$$

$$\mu_{h2}\mu_{h1}\begin{vmatrix} K_1 & -\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} \\ K_2 & \mu_2 \end{vmatrix} =$$
$$\mu_{h2}\mu_{h1}\left\{K_1\mu_2+K_2\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\right\}$$

Hence the solution is

$$L_1=\frac{\mu_{h2}\mu_{h1}\left\{K_1\mu_2+K_2\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\right\}}{\mu_{h1}\mu_{h2}\left\{\mu_1\mu_2-\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\right\}}= \frac{K_1\mu_2+K_2\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}}{\left\{\mu_1\mu_2-\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\right\}} \tag{94}$$

The solution for L_{h1}, can be obtained from

$$\begin{vmatrix} \mu_1 & -\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & K_1 & 0 & 0 & 0 \\ -\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & \mu_2 & K_2 & 0 & 0 & 0 \\ 0 & 0 & H_1 & 0 & 0 & 0 \\ 0 & 0 & H_2 & \mu_{h2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \end{vmatrix} =$$
$$\begin{vmatrix} \mu_1 & -\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & K_1 & 0 \\ -\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & \mu_2 & K_2 & 0 \\ 0 & 0 & H_1 & 0 \\ 0 & 0 & H_2 & \mu_{h2} \end{vmatrix} =$$
$$\mu_{h2}H_1\begin{vmatrix} \mu_1 & -\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} \\ -\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} & \mu_2 \end{vmatrix} =$$
$$\mu_{h2}H_1\left\{\mu_1\mu_2-\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\right\}$$

Hence the solution is

$$L_{h1}=\frac{\mu_{h2}H_1\left\{\mu_1\mu_2-\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\right\}}{\mu_{h1}\mu_{h2}\left\{\mu_1\mu_2-\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}\right\}}= \frac{H_1}{\mu_{h1}} \tag{95}$$

Mathematical Appendix B. Steady-state solutions of the general model

Banks in steady state maximize the following profit function:

$$\begin{aligned}\pi^i_\gamma &= \left[\left(1-\delta_\gamma\right)r_L-a_\gamma\right]L^i_\gamma+\left[\left(1-\delta_{h\gamma}\right)r_{Lh,\gamma}-a_\gamma\right]L^i_{h\gamma}+r_F F^i_\gamma+r_{G,\gamma}G^i_\gamma \\ &\quad + r_R R^i_\gamma - r_{D,\gamma}D^i_\gamma - r_E E^i_\gamma - \frac{\mu_\gamma}{2}\left(L^i\right)^2-\frac{\mu_{h\gamma}}{2}\left(L^i_{h\gamma}\right)^2-\frac{\omega}{2}\left(D^i_\gamma\right)^2 \\ &\quad -\frac{\omega}{2}\left(DD^i_\gamma\right)^2-\frac{\sigma}{2}\left(E^i_\gamma\right)^2-\frac{\phi_\gamma}{2}\left(F^i_\gamma\right)^2-\frac{\nu_\gamma}{2}\left(G^i_\gamma\right)^2-\frac{\tau_\gamma}{2}\left(R^i_\gamma\right)^2,\end{aligned} \tag{96}$$

s.t.

$$\left(1-\theta_\gamma\right)L^i_{\gamma,t}+\left(1-\theta_\gamma\right)L^i_{h\gamma,t}+G^i_{\gamma,t}+F^i_{\gamma,t}+R^i_{\gamma,t}=(1-q)DD^i_{\gamma,t}+D^i_{\gamma,t}. \tag{97}$$

with

$$E^i_\gamma=\theta_\gamma L^i_\gamma+\theta_\gamma L^i_{h\gamma}, \tag{98}$$

$$R^i_\gamma+RR^i_\gamma=qDD^i_\gamma. \tag{99}$$

$$r_L=-\eta\bar{r}_L\left(\overline{L}\right)^{-1}L+\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F+(\eta-\xi+1)\,\bar{r}_L, \tag{100}$$

$$L=-\overline{L}\left(\eta\bar{r}_L\right)^{-1}r_L+\xi\overline{L}\left(\eta\bar{r}_F\right)^{-1}r_F+\eta^{-1}\left(\eta-\xi+1\right)\overline{L}, \tag{101}$$

$$r_{Lh,\gamma} = -\eta_{h,\gamma}\overline{r_{Lh,\gamma}}\left(\overline{L}\right)^{-1} L_{h\gamma} + \xi\overline{r_{Lh}}\left(\overline{r_{G,\gamma}}\right)^{-1} r_{G,\gamma} + \left(\eta_{h,\gamma} - \xi + 1\right)\overline{r_{Lh,\gamma}} - , \tag{102}$$

$$L_{h\gamma} = -\overline{L_{h\gamma}}\left(\eta\overline{r_{Lh,\gamma}}\right)^{-1} r_{Lh,\gamma} + \xi\overline{L}h\left(\eta_{h,\gamma}\overline{r_{G,\gamma}}\right)^{-1} r_{G,\gamma} + \eta_{h,\gamma}^{-1}\left(\eta_{h,\gamma} - \xi + 1\right)\overline{L_{h\gamma}}, \tag{103}$$

$L^i_{h\gamma}$ is the loan quantity in the regional market γ, while L^i_γ is the quantity of the common-market loans. These linearized functions are obtained from

$$L = \left[\Sigma\left(r_L\right)^{-1}\left(r_F\right)^{\xi}\right]^{\frac{1}{\eta}},\ L_{\gamma h} = \left[\Sigma_{\gamma h}\left(r_{Lh,\gamma}\right)^{-1}\left(r_{G,\gamma}\right)^{\xi}\right]^{\frac{1}{\eta_{h,\gamma}}} \text{ and } L = L_1 + L_2 \text{ and } \overline{L} = \overline{L_1} + \overline{L_2}.$$

The central bank faces the following balance sheet constraint:

$$F^C + F_{CB} = RR_1 + RR_2 + R_1 + R_2, \tag{104}$$

where F^C is the amount of securities in the asset portfolio and F_{CB} represents the central bank's net repurchase agreements.

$$D_\gamma = \varepsilon_\gamma{}^{-1}(\varepsilon_\gamma + \alpha - 1)\overline{D} + \overline{D}\left(\varepsilon\overline{r_D}\right)^{-1} r_{D,\gamma} - \overline{D}\alpha\left(\varepsilon_\gamma\overline{r_{G,\gamma}}\right)^{-1} r_{G,\gamma}, \tag{105}$$

$$r_{D,\gamma} = \varepsilon_\gamma\bar{r}_{D,\gamma}\left(\overline{D}\right)^{-1} D_\gamma + \alpha\bar{r}_{D,\gamma}\left(\overline{r_{G,\gamma}}\right)^{-1} r_{G,\gamma} - (\varepsilon_\gamma + \alpha - 1)\bar{r}_{D,\gamma}, \tag{106}$$

These linearized functions are obtained from $D_\gamma = \left[Z_\gamma r_{D,\gamma}\left(r_F\right)^{-\alpha}\right]^{\frac{1}{\varepsilon}}$.

We assume that

$$r_{G\gamma} = r_F + \zeta_\gamma, \tag{107}$$

where ζ_γ is a region-specific risk premium.

The maximization problem is the following:

$$\begin{aligned}
\Lambda^i_\gamma = &\left\{\left(1-\delta_\gamma\right)\left[-\eta\overline{r_{L,\gamma}}\left(\overline{L}\right)^{-1}\left(L^i_\gamma + \hat{L}^i_\gamma\right) + \xi\overline{r_{L,\gamma}}\left(\bar{r}_F\right)^{-1} r_F + (\eta - \xi + 1)\bar{r}_L\right] - a_\gamma - \tau_\gamma\right\} L^i \\
&+ \left\{\left(1-\delta_{h\gamma}\right)\left[-\eta_{h,\gamma}\overline{r_{Lh,\gamma}}\left(\overline{L}\right)^{-1}\left(L^i_{h\gamma} + \hat{L}^i_{h\gamma}\right) + \xi\overline{r_{Lh,\gamma}}\left(\overline{r_G}\right)^{-1} r_{G,\gamma} + \left(\eta_{h,\gamma} + 1\right)\overline{r_{Lh,\gamma}}\right] r_{Lh,\gamma} - a_\gamma\right\} L^i_{h\gamma} + r_F F^i_\gamma + r_R R^i_\gamma \\
&+ r_{G,\gamma} G^i_\gamma - \left[\varepsilon_\gamma\bar{r}_{D,\gamma}\left(\overline{D}\right)^{-1}\left(D^i_\gamma + \hat{D}^i_\gamma\right) + \alpha\bar{r}_{D,\gamma}\left(\overline{r_G}\right)^{-1} r_{G,\gamma} - (\varepsilon_\gamma + \alpha - 1)\bar{r}_{D,\gamma}\right] D^i_\gamma - r_E(\theta_\gamma L^i_\gamma + \theta_\gamma L^i_{h\gamma}) - \frac{\mu_\gamma}{2}\left(L^i\right)^2 \\
&- \frac{\mu_{h\gamma}}{2}\left(L^i_{h\gamma}\right)^2 - \frac{\omega}{2}\left(D^i_\gamma\right)^2 - \frac{\omega}{2}\left[R^i_\gamma + RR^i_\gamma\left(1 - \frac{1}{q}\right)\right]^2 - \frac{\sigma}{2}\left(\theta_\gamma L^i_\gamma + \theta_\gamma L^i_{h\gamma}\right)^2 - \frac{\phi_\gamma}{2}\left(F^i_\gamma\right)^2 - \frac{\nu_\gamma}{2}\left(G^i_\gamma\right)^2 - \frac{\tau_\gamma}{2}\left(R^i_\gamma\right)^2 \\
&- \lambda^i_\gamma\left[\left(1-\theta_\gamma\right) L^i_{\gamma,t} + \left(1-\theta_\gamma\right) L^i_{h\gamma,t} + G^i_{\gamma,t} + F^i_{\gamma,t} + R^i_{\gamma,t} - (1-q)DD^i_{\gamma,t} - D^i_{\gamma,t}\right],
\end{aligned} \tag{108}$$

The first order conditions are the following:

$$\begin{aligned}\frac{\partial\Lambda^i_\gamma}{\partial L^i} &= \left(1-\delta_\gamma\right) r_L - a_\gamma - L^i\left(1-\delta_\gamma\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} \\ &-r_E\theta_\gamma - \mu L^i - \sigma\theta^2_\gamma L^i - \sigma\theta^2_\gamma L^i_{h\gamma} - (1-\theta_\gamma)\lambda_\gamma = 0.\end{aligned} \tag{109}$$

$$\begin{aligned}\frac{\partial\Lambda^i_\gamma}{\partial L^i_{h,\gamma}} &= \left(1-\delta_{h\gamma}\right) r_{Lh,\gamma} - a_\gamma - L^i_{h\gamma}\left(1-\delta_{h\gamma}\right)\eta_{h,\gamma}\overline{r_{Lh,\gamma}}\left(\overline{L}\right)^{-1} - r_E\theta_\gamma \\ &- \left(\mu_{h\gamma} + \sigma\theta^2_\gamma\right) L^i_\gamma - L^i_{h\gamma} - \sigma\theta^2_\gamma L^i - (1-\theta_\gamma)\lambda_\gamma = 0.\end{aligned} \tag{110}$$

$$\frac{\partial\Lambda^i_\gamma}{\partial F^i_\gamma} = r_F - \phi_\gamma F^i_\gamma - \lambda^i_\gamma = 0. \tag{111}$$

$$\frac{\partial\Lambda^i_\gamma}{\partial R^i_\gamma} = r_R - \omega R^i_\gamma\left(1 + RR^i_\gamma\left|1 - \frac{1}{q}\right|\right) - \tau R^i_\gamma - \lambda^i_\gamma \le 0. \tag{112}$$

$$\frac{\partial\Lambda^i_\gamma}{\partial G^i_\gamma} = r_{G,\gamma} - \nu_\gamma G^i_\gamma - \lambda^i_\gamma = 0 \tag{113}$$

$$\frac{\partial\Lambda^i_\gamma}{\partial D^i_\gamma} = -r_{D,\gamma} - \varepsilon_\gamma\bar{r}_{D,\gamma}\left(\overline{D}\right)^{-1} D^i_\gamma - \omega D^i_\gamma + \lambda^i_\gamma = 0. \tag{114}$$

$$\frac{\partial\Lambda^i_\gamma}{\partial\lambda^i_\gamma} = \left(1-\theta_\gamma\right) L^i_\gamma + \left(1-\theta_\gamma\right) L^i_{h\gamma} + G^i_\gamma + F^i_\gamma - D^i_\gamma = 0. \tag{115}$$

The first order conditions for common-market loans can be aggregated for the respective number of banks in each market as follows:

$$\begin{aligned}L_1 = n_1 L^i_1 &: \\ n_1\left(1-\delta_1\right) r_L - n_1 a_1 &- n_1 L^i_1\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1} \\ -n_1 r_E\theta_1 - n_1\left(\mu_1 + \sigma\theta^2_1\right) L^i_1 &- n_1\sigma\theta^2_1 L^i_{h1} - n_1(1-\theta_1)\lambda_1 = 0.\end{aligned} \tag{116}$$

$$L_2 = n_2 L_2^i : \quad n_2\left(1-\delta_2\right) r_L - n_2 a_2 - n_2 L_2^i \left(1-\delta_2\right) \eta \bar{r}_L \left(\overline{L}\right)^{-1} - n_2 r_E \theta_2 - n_2\left(\mu_2+\sigma\theta_2^2\right) n_2 L_2^i - n_2 \sigma \theta_2^2 L_{h2}^i - n_2(1-\theta_2)\lambda_2 = 0. \tag{117}$$

And given that$r_L = -\eta \bar{r}_L \left(\overline{L}\right)^{-1} L + \xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F + (\eta-\xi+1)\bar{r}_L$, it follows that

$$r_L = -\left(L_1+L_2\right)\eta \bar{r}_L \left(\overline{L}\right)^{-1} + \xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F + (\eta-\xi+1)\bar{r}_L \tag{118}$$

and

$$-L_1 \eta \bar{r}_L \left(\overline{L}\right)^{-1} = r_L + L_2 \eta \bar{r}_L \left(\overline{L}\right)^{-1} - \xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F - (\eta-\xi+1)\bar{r}_L \tag{119}$$

Hence since $L_1 = n_1 L_1^i$ and $L_2 = n_2 L_2^i$:

$$n_1\left(1-\delta_1\right) r_L - n_1 a_1 + \left(1-\delta_1\right)\left[r_L + L_2 \eta \bar{r}_L \left(\overline{L}\right)^{-1} - \xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F - (\eta-\xi+1)\bar{r}_L\right] - n_1 r_E \theta_1 - n_1\left(\mu_1+\sigma\theta_1^2\right) L_1^i - n_1 \sigma \theta_1^2 L_{h1}^i - n_1(1-\theta_1)\lambda_1 = 0. \tag{120}$$

$$n_2\left(1-\delta_2\right) r_L - n_2 a_2 + \left(1-\delta_2\right)\left[r_L + L_1 \eta \bar{r}_L \left(\overline{L}\right)^{-1} - \xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F - (\eta-\xi+1)\bar{r}_L\right] - n_2 r_E \theta_2 - n_2\left(\mu_2+\sigma\theta_2^2\right) L_2^i - n_2 \sigma \theta_2^2 L_{h2}^i - n_2(1-\theta_2)\lambda_2 = 0. \tag{121}$$

And,

$$\left(n_1+1\right)\left(1-\delta_1\right) r_L + \left(1-\delta_1\right)\eta \bar{r}_L \left(\overline{L}\right)^{-1} L_2 - \left(1-\delta_1\right)\left[\xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F + (\eta-\xi+1)\bar{r}_L\right] - \left(\mu_1+\sigma\theta_1^2\right) L_1 - \sigma\theta_1^2 L_{h1} - (1-\theta_1)\Lambda_1 - n_1 a_1 - n_1 r_E \theta_1 = 0. \tag{122}$$

$$\left(n_2+1\right)\left(1-\delta_2\right) r_L + \left(1-\delta_2\right)\eta \bar{r}_L \left(\overline{L}\right)^{-1} L_1 - \left(1-\delta_2\right)\left[\xi \bar{r}_L \left(\bar{r}_F\right)^{-1} r_F + (\eta-\xi+1)\bar{r}_L\right] - \left(\mu_2+\sigma\theta_2^2\right) L_2 - \sigma\theta_2^2 L_{h2}^i - (1-\theta_2)\Lambda_2 - n_2 a_2 - n_2 r_E \theta_2 = 0. \tag{123}$$

The first order conditions for regional lending in region 1 can be aggregated as follows:

$$L_{h1} = n_1 L_{h1}^i: \quad n_1\left(1-\delta_{h1}\right) r_{Lh,1} - n_1 a_1 - n_1 L_{h1}^i \left(1-\delta_{h\gamma}\right)\eta_{h1} \overline{r_{Lh,\gamma}} \left(\overline{L}\right)^{-1} - n_1 r_E \theta_1 - \left(\mu_{h1}+\sigma\theta_1^2\right) n_1 L_{h1}^i - \sigma\theta_1^2 L_1 - (1-\theta_1) n_1 \lambda_1 = 0. \tag{124}$$

And given that $r_{Lh,\gamma} = -\eta_{h,\gamma} \overline{r_{Lh,\gamma}} \left(\overline{L}\right)^{-1} L_{h\gamma} + \xi \overline{r_{Lh}} \left(\overline{r_{G,\gamma}}\right)^{-1} r_{G,\gamma} + \left(\eta_{h,\gamma}-\xi+1\right)\overline{r_{Lh,\gamma}}$, it follows that

$$-\eta_{h,\gamma} \bar{r}_{Lh1} \left(\overline{L}\right)^{-1} L_{h1} = r_{Lh1} - \xi \overline{r_{Lh}} \left(\bar{r}_{G1}\right)^{-1} r_{G1} - \left(\eta_{h,\gamma}-\xi+1\right)\bar{r}_{Lh1} \tag{125}$$

Hence since $L_{h1} = n_1 L_{h1}^i$,

$$(n_1+1)\left(1-\delta_{h1}\right) r_{Lh,1} - n_1 a_1 - \left(1-\delta_{h1}\right)\left[\xi \bar{r}_{Lh1} \left(\bar{r}_{G1}\right)^{-1} r_{G1} + \left(\eta_{h1}-\xi+1\right)\bar{r}_{Lh1}\right] - n_1 r_E \theta_1 - \left(\mu_{h1}+\sigma\theta_1^2\right) L_{h1} - \sigma\theta_1^2 L_1 - (1-\theta_1)\Lambda_1 = 0. \tag{126}$$

And similarly in region 2

$$(n_2+1)\left(1-\delta_{h2}\right) r_{Lh2} - n_2 a_2 - \left(1-\delta_{h2}\right)\left[\xi \overline{r_{L2}} \left(\bar{r}_{G2}\right)^{-1} r_{G2} + \left(\eta_{h2}-\xi+1\right)\bar{r}_{Lh2}\right] - n_2 r_E \theta_2 - \left(\mu_{h2}+\sigma\theta_2^2\right) L_{h2} - \sigma\theta_2^2 L_2 - \left(1-\theta_2\right)\Lambda_2 = 0. \tag{127}$$

These conditions differ from the equivalent ones in the common loan market because of the absence of the foreign spillover term.

The first order condition for securities can be aggregated as:

$$G_1 = n_1 G_1^i: \quad n_1 r_{G1} - n_1 \nu_1 G_1^i - n_1 \lambda_1^i = 0, \tag{128}$$

The other first order conditions can be aggregated as follows:

$$F_1 = n_1 F_1^i: \quad n_1 r_F - n_1 \phi_1 F_1^i - n_1 \lambda_1^i = 0 \tag{129}$$

$$D_1 = n_1 D_1^i: \quad -n_1 r_{D,1} - n_1 \varepsilon_{h1} \overline{r_{D,1}} \left(\overline{D}\right)^{-1} D_1^i - n_1 \omega D_1^i + n_1 \lambda_1^i = 0. \tag{130}$$

$$\Lambda_1 = n_1 \lambda_1^i: \quad n_1\left(1-\theta_1\right) L_1^i + n_1\left(1-\theta_1\right) L_{1h}^i + n_1 G_1^i + n_1 F_1^i - n_1 D_1^i = 0. \tag{131}$$

After substituting the value of the demand for loans and the supply of deposits, for, respectively, $n_\gamma L^i_{h\gamma}$, and $n_\gamma D^i_\gamma$, these conditions can be rewritten to obtain the aggregate quantities for each region as follows:

$$\begin{aligned}&\left(\mu_1+\sigma\theta_1^2\right)L_1+\sigma\theta_1^2 L_{h1}+\left(1-\theta_1\right)\Lambda_1-\left(1-\delta_1\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}L_2=\\&\quad\left(n_1+1\right)\left(1-\delta_1\right)r_L-n_1a_1-\left(1-\delta_1\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F-n_1r_E\theta_1\\&\qquad-\left(1-\delta_1\right)\left(\eta-\xi+1\right)\bar{r}_L\end{aligned}\tag{132}$$

$$\begin{aligned}&\left(\mu_2+\sigma\theta_2^2\right)L_2+\sigma\theta_2^2 L_{h2}+\left(1-\theta_2\right)\Lambda_2-\left(1-\delta_2\right)\eta\bar{r}_L\left(\overline{L}\right)^{-1}L_1=\\&\quad\left(n_2+1\right)\left(1-\delta_2\right)r_L-n_2a_2-\left(1-\delta_2\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}r_F-n_2r_E\theta_2\\&\qquad-\left(1-\delta_2\right)\left(\eta-\xi+1\right)\bar{r}_L.\end{aligned}\tag{133}$$

$$\left(\mu_{h1}+\sigma\theta_1^2\right)L_{h1}+\sigma\theta_1^2L_1+(1-\theta_1)\Lambda_1=(n_1+1)\left(1-\delta_{h1}\right)r_{Lh,1}-n_1a_1-n_1r_E\theta_1-\left(1-\delta_{h1}\right)\left[\xi\bar{r}_{Lh1}\left(\bar{r}_{G1}\right)^{-1}r_{G1}+\left(\eta_{h1}-\xi+1\right)\bar{r}_{Lh1}\right]\tag{134}$$

$$\left(\mu_{h2}+\sigma\theta_2^2\right)L_{h2}+\sigma\theta_2^2L_2+(1-\theta_2)\Lambda_2=(n_2+1)\left(1-\delta_{h2}\right)r_{Lh,2}-n_2a_2-n_2r_E\theta_2-\left(1-\delta_{h2}\right)\left[\xi\bar{r}_{Lh2}\left(\bar{r}_{G2}\right)^{-1}r_{G2}+\left(\eta_{h2}-\xi+1\right)\bar{r}_{Lh2}\right].\tag{135}$$

Similarly, from

$$-n_1r_{D,1}-n_1\varepsilon_{h1}\overline{r_{D,1}}\left(\overline{D}\right)^{-1}D_1^i-n_1\omega D_1^i+n_1\lambda_1^i=0$$

we get after aggregation

$$-n_1r_{D,1}-\varepsilon_{h1}\overline{r_{D,1}}\left(\overline{D}\right)^{-1}D_1-\omega D_1+\Lambda_1=0$$

and since

$$r_{D,\gamma}=\varepsilon_\gamma\bar{r}_{D,\gamma}\left(\overline{D}\right)^{-1}D_\gamma+\alpha\bar{r}_{D,\gamma}\left(\bar{r}_{G,\gamma}\right)^{-1}r_{G,\gamma}-(\varepsilon_\gamma+\alpha-1)\bar{r}_{D,\gamma},\ \text{and hence}$$

$$\varepsilon_\gamma\bar{r}_{D,\gamma}\left(\overline{D}\right)^{-1}D_\gamma=r_{D,\gamma}-\alpha\bar{r}_{D,\gamma}\left(\bar{r}_{G,\gamma}\right)^{-1}r_{G,\gamma}+(\varepsilon_\gamma+\alpha-1)\bar{r}_{D,\gamma},$$

we get

$$-\omega D_1+\Lambda_1=\left(n_1+1\right)r_{D1}-\alpha\bar{r}_{D1}\left(\bar{r}_{G1}\right)^{-1}r_{G1}+(\varepsilon_1+\alpha-1)\bar{r}_{D1}.\tag{136}$$

$$-\omega D_2+\Lambda_2=\left(n_2+1\right)r_{D2}-\alpha\bar{r}_{D2}\left(\bar{r}_{G2}\right)^{-1}r_{G2}+(\varepsilon_2+\alpha-1)\bar{r}_{D2}.\tag{137}$$

$$v_1G_1+\Lambda_1=n_1r_{G1},\tag{138}$$

$$v_2G_2+\Lambda_2=n_2r_{G2}.\tag{139}$$

Which together with

$$\phi_1F_1+\Lambda_1=n_1r_F,\tag{140}$$

$$\phi_2F_2+\Lambda_2=n_2r_F,\tag{141}$$

and the solution for reserves, which since $RR_\gamma=RR^i_\gamma$ can be written as

$$\left[\tau+\omega\left(1+RR_1\left|1-\frac{1}{q}\right|\right)\right]R_1+\Lambda_1\geq n_1r_R.\tag{142}$$

$$\left[\tau+\omega\left(1+RR_2\left|1-\frac{1}{q}\right|\right)\right]R_2+\Lambda_2\geq n_2r_R.\tag{143}$$

and

$$\left(1-\theta_1\right)L_1+\left(1-\theta_1\right)L_{h1}+G_1+F_1+R_1-(1-q)DD_1-D_1=0,\tag{144}$$

$$\left(1-\theta_2\right)L_2+\left(1-\theta_2\right)L_{h2}+G_2+F_2+R_2-(1-q)DD_2-D_2=0,\tag{145}$$

form a system of twelve equations in the ten unknown quantities L_1, L_2 L_{h1}, L_{h2}, G_1, G_2, F_1, F_2, D_1, D_2 plus the multipliers Λ_1 and Λ_2. The system can be reduced to ten equations in ten unknowns by substituting the value of the multipliers: $\Lambda_1=n_1r_F-\phi_1F_1$

and $\Lambda_2 = n_2 r_F - \phi_2 F_2$.

$$\begin{aligned}&\left(\mu_1+\sigma\theta_1^2\right)L_1+\sigma\theta_1^2 L_{h1}-\left(1-\theta_1\right)\phi_1 F_1-\left(1-\delta_1\right)\eta\bar{r}_L\left(\bar{L}\right)^{-1}L_2=\\&\left(n_1+1\right)\left(1-\delta_1\right)r_L\\&-n_1 a_1-\left\{n_1\left(1-\theta_1\right)+\left(1-\delta_1\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}\right\}r_F-n_1 r_E\theta_1-\left(1-\delta_1\right)\left(\eta-\xi+1\right)\bar{r}_L.\end{aligned} \tag{146}$$

$$\begin{aligned}&\left(\mu_2+\sigma\theta_2^2\right)L_2+\sigma\theta_2^2 L_{h2}-\left(1-\theta_2\right)\phi_2 F_2-\left(1-\delta_2\right)\eta\bar{r}_L\left(\bar{L}\right)^{-1}L_1=\\&\left(n_2+1\right)\left(1-\delta_2\right)r_L\\&-n_2 a_2-\left\{n_2(1-\theta_2)+\left(1-\delta_2\right)\xi\bar{r}_L\left(\bar{r}_F\right)^{-1}\right\}r_F-n_2 r_E\theta_2-\left(1-\delta_2\right)\left(\eta-\xi+1\right)\bar{r}_L,\end{aligned} \tag{147}$$

$$\begin{aligned}\left(\mu_{h1}+\sigma\theta_1^2\right)L_{h1}+\sigma\theta_1^2 L_1-(1-\theta_1)\phi F_1&=(n_1+1)\left(1-\delta_{h1}\right)r_{Lh,1}\\&-(1-\theta_1)n_1 r_F-n_1 a_1-n_1 r_E\theta_1-\left(1-\delta_{h1}\right)\left[\xi\bar{r}_{Lh1}\left(\bar{r}_{G1}\right)^{-1}r_{G1}+\left(\eta_{h1}-\xi+1\right)\bar{r}_{Lh1}\right].\end{aligned} \tag{148}$$

$$\begin{aligned}\left(\mu_{h2}+\sigma\theta_2^2\right)L_{h2}+\sigma\theta_2^2 L_2-(1-\theta_2)\phi F_2&=(n_2+1)\left(1-\delta_{h2}\right)r_{Lh,2}-\left(1-\theta_2\right)n_2 r_F-n_2 a_2-n_2 r_E\theta_2-\left(1-\delta_{h2}\right)\\&\times\left[\xi\bar{r}_{Lh2}\left(\bar{r}_{G2}\right)^{-1}r_{G2}+\left(\eta_{h2}-\xi+1\right)\bar{r}_{Lh2}\right].\end{aligned} \tag{149}$$

$$\phi F_1+\omega D_1=n_1 r_F-\left(n_1+1\right)r_{D,1}+\alpha\bar{r}_{D1}\left(\bar{r}_{G1}\right)^{-1}r_{G1}-(\epsilon_1+\alpha-1)\bar{r}_{D1}, \tag{150}$$

$$\phi_2 F_2+\omega D_1=n_2 r_F-\left(n_2+1\right)r_{D,2}+\alpha\bar{r}_{D2}\left(\bar{r}_{G2}\right)^{-1}r_{G2}-(\epsilon_2+\alpha-1)\bar{r}_{D2}, \tag{151}$$

$$G_1=\frac{\phi_1}{v_1}F_1+\frac{n_1}{v_1}(r_{G1}-r_F), \tag{152}$$

$$G_2=\frac{\phi_2}{v_2}F_2+\frac{n_2}{v_2}(r_{G2}-r_F), \tag{153}$$

$$R_1\geq\frac{n_1 r_R-(n_1 r_F-\phi_1 F_1)}{\left[\tau+\omega\left(1+RR_1\left|1-\frac{1}{q}\right|\right)\right]}. \tag{154}$$

$$R_2\geq\frac{n_2 r_R-(n_2 r_F-\phi_2 F_2)}{\left[\tau+\omega\left(1+RR_2\left|1-\frac{1}{q}\right|\right)\right]}. \tag{155}$$

$$\left(1-\theta_1\right)L_1+\left(1-\theta_1\right)L_{h1}+G_1+F_1+R_1-(1-q)DD_1-D_1=0, \tag{156}$$

$$\left(1-\theta_2\right)L_2+\left(1-\theta_2\right)L_{h2}+G_2+F_2+R_2-(1-q)DD_2-D_2=0, \tag{157}$$

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