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ABSTRACT

A magnetic proton recoil (MPR) neutron spectrometer is being designed for SPARC, a high magnetic field ($B_T = 12$ T), compact ($R_0 = 1.85$ m, $a = 0.57$ m) tokamak currently under construction in Devens, MA, USA. MPR neutron spectrometers are versatile tools for making high fidelity *ab initio* calibrated measurements of fusion neutron flux spectra and have been used to infer fusion power, ion temperature, fuel ion ratio, and suprathermal fuel populations at several high performance fusion experiments. The performance of an MPR neutron spectrometer is in large part determined by the design of the magnetic field, which disperses and focuses recoil protons. This article details the ion optical design of a high-resolution MPR neutron spectrometer, including the amelioration of image aberrations due to nonlinear effects. An optimized design is presented that achieves ion optical energy resolution $\delta E/E < 1\%$ and focal plane properties that enable straightforward integration with the hodoscope detector array.

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I. INTRODUCTION

Neutrons produced by fusion reactions present an important diagnostic signal for studying the physics and performance of fusion reactors. In particular, the energy spectrum of neutron emissions from Deuterium–Deuterium (DD) and Deuterium–Tritium (DT) fusion reactions encodes useful kinetic information about the fuel.¹ The Magnetic Proton Recoil (MPR) technique is a powerful method that leverages the well understood physics of elastic nuclear scattering and charged particle transport through magnetic fields to make high-resolution calibrated measurements of neutron flux spectra.

The MPR technique was first proposed for studying fusion neutron emissions in 1992² and was experimentally pioneered at the JET tokamak to make detailed studies of the DT neutron production

spectrum.³ Since then, the MPR technique has been successfully implemented at the OMEGA and NIF laser facilities.⁴ MPR based measurements of neutron emissions from these fusion experiments have been used to deduce many important plasma parameters, including total fusion rate,⁵ ion temperature,⁶ bulk plasma flows,⁷ fuel ion concentrations,⁸ and non-thermal fusion.⁹

The operating principles of the technique are summarized schematically in Fig. 1. Collimated neutrons (1A) emitted from the fusing plasma impinge upon a thin, <250 μm , conversion foil (1B), where they elastically scatter on protons. An aperture (1C) selects protons scattered into a small, <4 msr, solid angle about the forward direction, forming a diffuse proton beam. This beam then traverses a magnetic beamline, which focuses the protons and disperses them according to their energy. The dispersed protons then strike an array of detectors (1F), called a hodoscope,¹⁰ which records

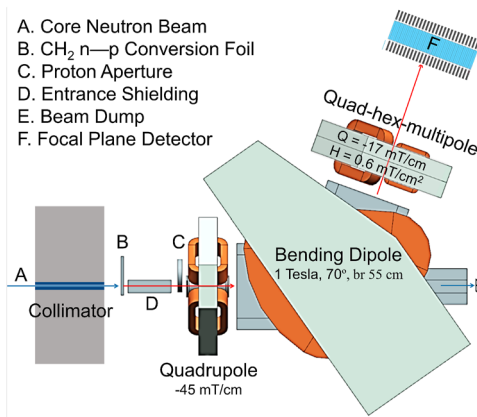


FIG. 1. Schematic drawing of the MPR spectrometer for SPARC with major components labeled. The magnetic field parameters listed here correspond to a 14 MeV reference proton energy.

the spatial distribution of protons. The measured spatial distribution of the protons is directly related to the proton energy spectrum by the dispersion of the beamline (see Fig. 5). The proton spectrum is related to the incident neutron energy spectrum by the aperture geometry and stopping power of the conversion foil. The response function can be directly calculated from first principles using accurately measured quantities, constituting an *ab initio* calibration of the instrument.^{2,4}

The overall neutron energy resolution is set by three components: the conversion process, which broadens and downshifts the proton spectrum relative to the neutron spectrum; the ion optical resolution given by Eq. (A10); and the finite hodoscope detector channel width, which sets the minimum resolvable scale length. The system is held under vacuum better than 0.1 mTorr in order to minimize the broadening of the proton spectrum via interactions with air molecules.

The next generation of MPR spectrometers is currently being designed to expand the capabilities and improve the quality of neutron spectrum measurements as fusion experiments begin to probe the burning plasma regime.¹¹ This work describes the design of the ion optical magnet system for an MPR spectrometer for the SPARC tokamak. In Sec. II, the spectrometer is contextualized at SPARC, and the current design is summarized. Section III describes the ion

optical design process in detail. The ion optical performance is discussed in Sec. IV. A brief description of the ion optical formalism is given in the Appendix.

II. MPR NEUTRON SPECTROMETER AT SPARC

SPARC is a high magnetic field ($B_T = 12$ T), compact ($R_0 = 1.85$ m, $a = 0.57$ m) tokamak designed to demonstrate net-energy fusion in a magnetically confined plasma and study burning plasmas.¹² A suite of neutron diagnostics plays an important role in realizing these goals.¹³ The MPR spectrometer constitutes the midplane line of sight of the poloidal neutron emission camera. Its viewing geometry is shown in Fig. 2. The MPR will be located outside the tokamak hall behind the 2.5 m thick concrete wall, which serves as the primary radiation shield for the spectrometer, although additional shielding is planned to further reduce background neutron and gamma fluxes. A 3 cm diameter collimator through the wall defines the field of view of the plasma. Except for the diagnostic port's thin vacuum flange and its own vacuum vessel, the view of SPARC's core plasma is unobstructed. Raj shows a rendering of the SPARC tokamak hall and neutron lab in these proceedings.¹³

The range of conditions expected across SPARC's experimental campaigns imposes several performance requirements on the sensitivity and energy resolution of the spectrometer. The absolute sensitivity requirements correspond to measurements of DT fusion power between 100 kW and 140 MW. The required resolution is set by measurement of the core ion temperature $T_i > 4$ keV via the thermal broadening of the primary neutron peak. Based on Brysk's Gaussian approximation for thermonuclear neutron spectra,¹⁴ this requirement implies an energy resolution capable of resolving a width of 350 keV for DT plasmas and 166 keV for DD plasmas.

This spectrometer is designed to maximize flexibility to meet its performance requirements. Electromagnets can tune the energy sensitivity of the instrument to cover the range from 2 to 20 MeV while preserving the relative ion optical performance, enabling high quality measurement of DD ($E_n \approx 2.45$ MeV) and DT ($E_n \approx 14.1$ MeV) neutron emissions. There is an inherent tension between the sensitivity and resolution of MPR spectrometers. Increasing the conversion efficiency necessitates a larger, thicker foil and/or a wider aperture, while improving the resolution calls for the opposite. To resolve this, mechanisms are being designed that will enable shot-to-shot selection of conversion foil and aperture. The design of the hodoscope detectors is critical to ensure accurate measurement of the proton distribution in the focal plane. Dalla Rosa

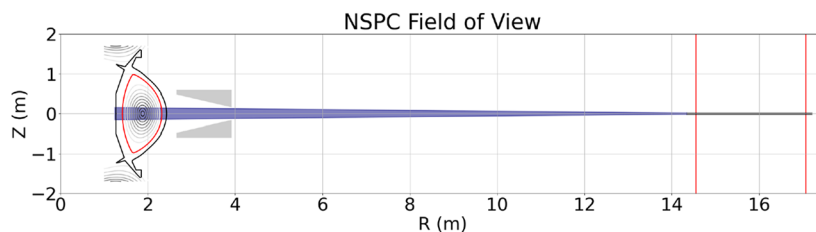


FIG. 2. Neutron spectrometer field of view defined by the 3 cm diameter collimator through the tokamak hall wall (red lines). The field of view admits neutrons with a maximum angle of 0.24° . The in-port shielding (in gray) has been designed to accommodate this field of view, giving a nearly unobstructed view of the core plasma out to $|Z| < 17$ cm.

describes detector prototype experiments for the hodoscope in these proceedings.¹⁰

III. ION OPTICAL DESIGN

The design of the magnet system was conducted using the ion optical formalism shown in the Appendix,¹⁵ and calculations were carried out using the differential algebraic Taylor map code COSY INFINITY.¹⁶

The conversion foil and aperture geometry define the initial size and divergence of the proton beam, which determines the size of the magnets. The design presented here considers a 1.5 cm radius foil and a 1.75 cm radius proton aperture separated by 50 cm. This geometry constraints the accepted rays to have $x_i, y_i < 1.5$ cm and $\theta_{xi}, \theta_{yi} < 3.7^\circ$.

A three magnet beamline consisting of a quadrupole, a dipole, and a multipole (quadrupole + hexapole) has been optimized through analysis of transfer map elements. The Taylor map formalism is leveraged by finding a first order configuration that meets the requirements, increasing the fidelity of the calculation, and applying corrections order by order.

A. First order design

The most important constraint on the first order design is to provide an ion optical focus. Restricting attention to the parameter

subspace that forms images, the system has been optimized to maximize the resolving power of the spectrometer [Eq. (A3)], subject to the saturation limits of the magnet iron and the size and power limits of the neutron lab.

The first magnet, a vertically focusing quadrupole, is placed 10 cm behind the aperture. Vertical focusing has two benefits: the vertical gap of the dipole is reduced without clipping the proton beam, which reduces the power requirement while maintaining full beam transmission, and it increases the horizontal dispersion of the beam, which improves the resolving power. The quadrupole has a bore radius of 6 cm.

The dipole is located 60 cm away from the exit of the quadrupole and deflects particles of the reference energy through a 70° , 55 cm radius arc. The field strength determines the magnetic rigidity of the reference particle; tuning the field to 0.98 T selects 14 MeV as the reference proton energy. The dipole has an 8 cm vertical gap and a 40 cm wide region of homogeneous magnetic field. Dipoles are inherently focusing in the dispersive plane and have no first order effect on vertical motion;¹⁵ however, the shapes of the pole faces provide additional degrees of freedom with which to tailor the ion optical trajectories in both directions. The slope of each face acts as an ion optical quadrupole.

A combined quadrupole/hexapole multipole is located 40 cm behind the exit of the dipole. The quadrupole field is vertically focusing to control the vertical image size, enabling shorter hodoscope

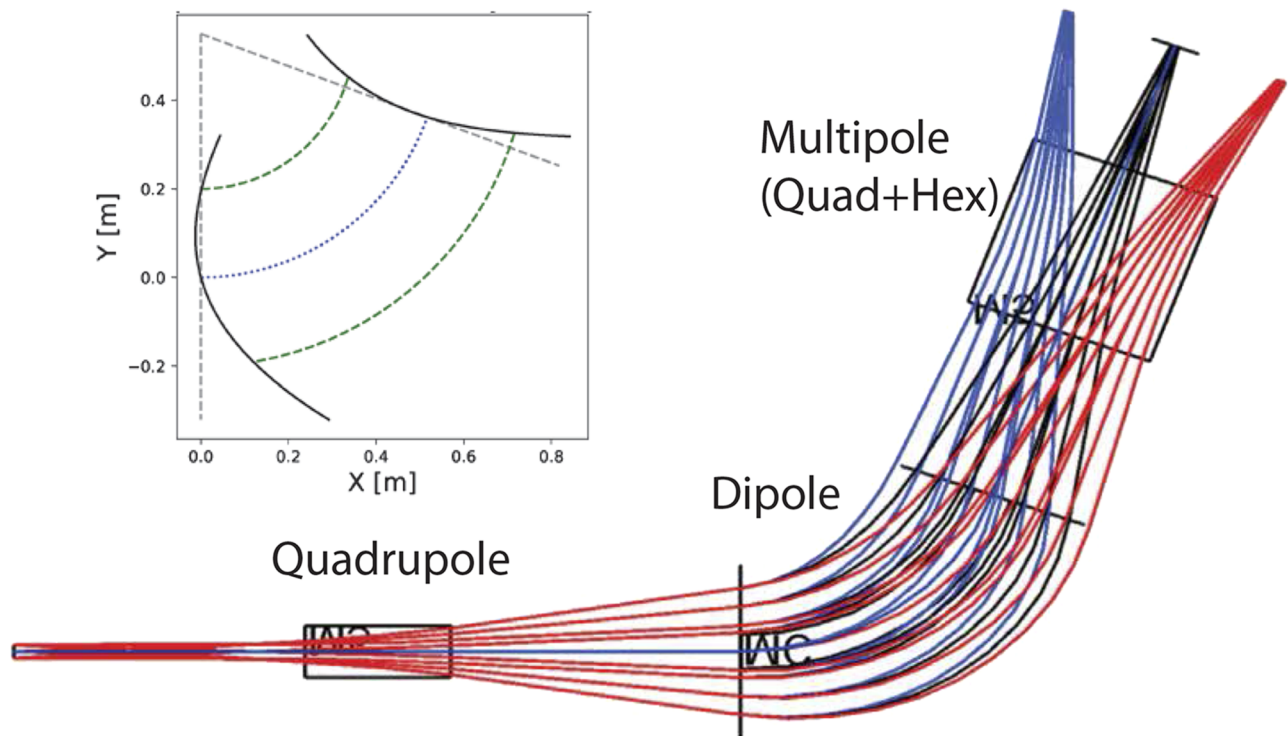


FIG. 3. First order trajectories projected onto the dispersive plane. Colors correspond to proton trajectories with different energies (red = $E_0 + 25\%$, black = E_0 , and blue = $E_0 - 25\%$). The ion optical resolving power is 81 ($\delta E/E = 1.2\%$). The inset shows the third order optimized dipole face shapes (solid black) and region of homogeneous field (dashed green).

detector units, which reduces background counts. To simultaneously observe the greatest range of energies while being power efficient, the multipole is designed to have an oblate bore to match the dimension of the dispersed proton beam. Modeled as an ellipse, the multipole has a horizontal major radius of 20 cm and a vertical minor radius of 10 cm. This multipole design gives an energy bite of $\pm 25\%$ about the central energy. The focal plane is 30 cm beyond the exit of the multipole.

The distances between elements were chosen based on manufacturing constraints on the size of the magnet coils and iron yokes, as well as the observation that the magnification roughly scales with the ratio of the length after the dipole to the length before the dipole. As the system's performance is sensitive to each parameter, the magnet parameters were determined iteratively by making small adjustments, refitting to find focus, and comparing the new performance to the design goal. Figure 3 shows first order optimized trajectories through the system.

B. Correction of aberrations

Including higher order terms in the calculation of the transfer map reveals the presence of aberrations that degrade the resolving power. Figure 4(a) shows a third order aberration as an example. The strategy is to ameliorate the aberrations order-by-order with multipole fields of the corresponding order. As is expected for a well behaved Taylor expansion, the higher order terms will become less and less important. This design is found to only require corrections up to the third order before the higher order aberrations no longer significantly contribute.

1. Second order corrections

The second order calculation of the transfer map includes coupling between phase space coordinates and introduces nonlinear distortions of the first order image and focal plane. The aberrations induced by second order terms of the transfer map are corrected by applying appropriate hexapole field effects, either through multipoles or by introducing curvature to the dipole faces.

One second order aberration was a 63° rotation of the focal plane away from perpendicular incidence. This chromatic aberration [Eq. (A8)] is controlled by the coupling of the initial angle and energy of protons in the final position and is corrected by hexapole effects following the dispersive action of the dipole. The

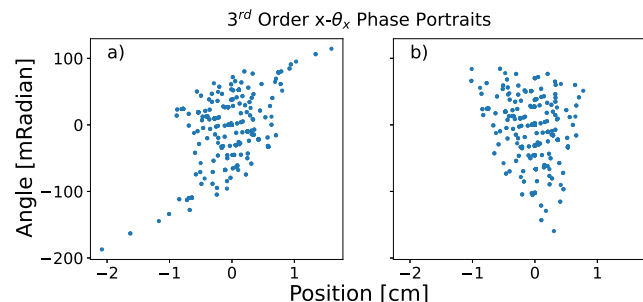


FIG. 4. Third order $x - \theta_x$ phase portraits: (a) Uncorrected cubic angular aberration, identified by the clear S-shape; and (b) corrected phase portrait lacking any obvious structure. Notice the reduction in dispersive image size from 6 to 1.5 cm. In addition, the focal plane radius of curvature is increased from 48 to 60 cm.

dipole exit face curvature and multipole's hexapole component are subsequently used to rotate the focal plane to normal incidence (0.13°).

The hexapole corrections used to rotate the focal plane exacerbated other aberrations. The largest of these was the quadratic angular aberration ($x|\theta_x\theta_x$). The entrance face curvature of the dipole is used to correct this aberration. The correction to the focal plane tilt also introduces a strong vertical chromatic aberration that causes the images formed by low energy protons to be shorter and wider and the images formed by high energy protons to be taller and thinner [see Fig. 5(b)]. Without adding another magnet, this aberration could not be corrected without sacrificing the image quality in the dispersive direction. As in the first order calculation, the optimum parameter values were found iteratively.

2. Third order corrections

Third order calculations introduce cubic nonlinearities and induce focal plane curvature [Eq. (A9)]. These aberrations were corrected by appropriate octupole field contributions provided by cubic shaping of the dipole faces. Figure 4 shows the uncorrected and corrected third order $x - \theta_x$ phase portraits, and the inset in Fig. 3 shows the resulting optimized third order dipole shape.

After making these third order corrections, it was found that higher order aberrations did not significantly degrade the ion optical performance of the system.

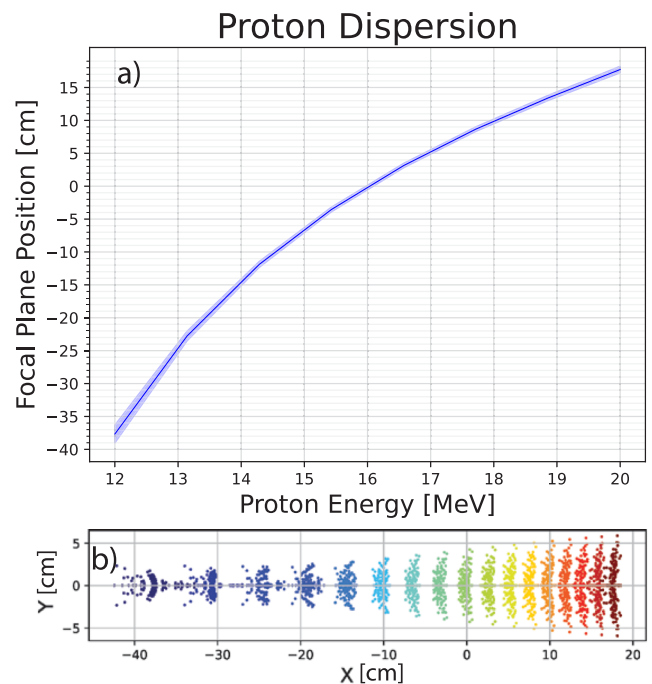


FIG. 5. (a) Proton energy dispersion for 16 MeV central energy. The shaded blue region shows the FWHM of Monte Carlo proton images and is 0.8 cm at 14 MeV, corresponding to a resolution of 127 keV. (b) Characteristic monoenergetic proton images spaced by 500 keV over the same energy range. 14 MeV protons land at $x = -15$ cm in this setting.

IV. ION OPTICAL PERFORMANCE

The performance of the spectrometer was assessed using the new open-sourced toolkit MPRTools,¹⁷ which will be described more fully in a future publication. MPRTools implements many useful functions for quantifying and visualizing the performance of MPR systems. In the following analyses, a fifth order transfer map generated by COSY INFINITY is used.

The toolkit implements a Monte Carlo procedure to generate a realistic ensemble of protons from user defined incident neutron spectra using ENDF differential scattering cross section data.¹⁸ Applying the transfer map produces realistic spatial proton distributions in the focal plane. The full width at half maximum (FWHM) of monoenergetic distributions can be used to compute the resolution according to Eq. (A10). The central ion optical resolution is computed to be 0.8% relative to the reference energy and is maintained as the reference energy is varied. The electromagnets can be tuned to any reference energy up to 16 MeV, at which point the dipole begins to saturate. The spectrometer measures protons with energies within 25% of the reference energy. Table I describes the ion optical resolution and measured energy range of the spectrometer for three reference energies of interest. The performance is more than sufficient to meet the resolution requirement described in Sec. II.

Figure 5(a) shows that the ion optics are heteroskedastic with energy. The FWHM increases for energies away from the reference, with the image broadening being more dramatic at the low end of the energy range. Figure 5(b) shows a series of monoenergetic proton images and reveals that the variation in FWHM is a result of the vertical chromatic aberration discussed in Sec. III B 1. This aberration could not be corrected without sacrificing resolution or increasing the system's complexity and was determined to be an acceptable trade-off from an integration perspective.

A. System integration

These analyses do not consider the broadening of the proton spectrum relative to the incident neutron spectrum, which must be studied in detail to determine the overall energy resolution of the spectrometer. Future work will find optimal combinations of foil radius, foil thickness, and aperture radius that maximize the count rate for a particular total energy resolution.

The focal plane of the ion optics has been carefully tailored to ease integration with the hodoscope detector array.¹⁰ The vertical image size was kept <10 cm. The focal plane was rotated to normal incidence with the optical axis, reducing the amount of crosstalk between neighboring detector channels and shortening the overall length of the focal plane. The curvature of the focal plane was also reduced to improve the coupling of proton trajectories with the detectors.

TABLE I. Ion optical properties when tuned to proton energies appropriate for the study of DD, DT, and non-thermal neutron emissions.

E_0 (MeV)	δE (MeV)	$E_{\min}$ (MeV)	$E_{\max}$ (MeV)
2.4	0.020	1.5	3.0
14.1	0.112	10.6	17.6
16	0.128	12	20

V. CONCLUSIONS

The ion optical design of the electromagnet system discussed in this paper meets its performance requirements. The design of the magnet system was conducted using COSY INFINITY to find a linear design point and then systematically address the higher order aberrations to maximize ion optical performance. The magnet system allows a tunable choice of reference energy ranging from 2 to 20 MeV and views protons with an energy of $\pm 25\%$ about the chosen reference energy. The ion optical energy resolution is 0.8% at the reference energy, enabling high resolution measurements of the neutron production spectrum of SPARC plasmas.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

S. Mackie: Conceptualization (lead); Formal analysis (lead); Investigation (lead); Software (lead); Visualization (lead); Writing – original draft (lead). **C. W. Wink:** Conceptualization (equal); Investigation (supporting); Writing – review & editing (supporting). **M. Dalla Rosa:** Methodology (supporting); Writing – review & editing (supporting). **G. P. A. Berg:** Formal analysis (equal); Investigation (equal); Methodology (equal); Software (equal); Supervision (equal); Writing – review & editing (supporting). **J. L. Ball:** Conceptualization (supporting); Visualization (supporting). **X. Wang:** Writing – review & editing (supporting). **J. Carmichael:** Investigation (supporting). **R. A. Tinguely:** Conceptualization (equal); Methodology (equal); Project administration (equal); Supervision (equal); Writing – review & editing (equal). **D. Rigamonti:** Methodology (supporting). **M. Tardocchi:** Methodology (supporting); Supervision (supporting). **P. Raj:** Project administration (equal); Visualization (equal); Writing – review & editing (equal). **J. Frenje:** Conceptualization (equal); Supervision (equal); Writing – review & editing (equal). **J. Rice:** Project administration (equal); Supervision (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

APPENDIX: ION OPTICAL FORMALISM

Ion optics comprise a charged particle transport formalism developed for the analysis of charged particle beamlines.¹⁵ Particles are described by phase space vectors defined relative to a reference particle's trajectory. x and θ_x describe the particle's horizontal (dispersive) displacement and angle away from the reference trajectory. y and θ_y describe the displacement and angle vertically. The deviation in the particle's energy is denoted by δE .

A transfer map, $\mathbf{T}$, is an operator that maps particle vectors from one point along their trajectories to another,

$$(x, \theta_x, y, \theta_y, \delta_E)_f = \mathbf{T}[(x, \theta_x, y, \theta_y, \delta_E)_i]. \quad (\text{A1})$$

A Taylor map is constructed as a set of multivariate Taylor polynomials for each phase space coordinate. The Taylor map of a given set of magnets may be explicitly calculated using numerical methods with high accuracy.¹⁹

1. Linear theory

First order ion optical calculations include only dipole and quadrupole magnetic field effects and are linear in phase space coordinates. The first order elements of $\mathbf{T}$, which determine x , the displacement of particles in the dispersive direction, are of particular importance for determining the performance of an ion optical spectrometer,

$$x_f = \mathbf{T}_1[(x, \theta_x, y, \theta_y, \delta_E)_i] = (x|x)x_i + (x|\theta)\theta_i + (x|\delta_E)\delta_{Ei}. \quad (\text{A2})$$

The locus of points where $(x|\theta_x) = 0$ defines the focal plane of the system. $(x|x)$ is the dispersive magnification and determines the size of the image in the focal plane. $(x|\delta_E)$ is the linear energy dispersion. The ion optical resolving power of a focused system is a linear figure of merit given by

$$R_0 = \frac{(x|\delta_E)}{2r_{\text{foil}}(x|x)}. \quad (\text{A3})$$

In first order calculations, the horizontal and vertical motions are decoupled: $(x|y) = (x|\theta_y) = (y|\theta_x) = 0$.

2. Nonlinear theory

Higher order calculations a Taylor map introduces nonlinear couplings between phase space coordinates that distort the image and modify the dispersion. The first nine terms of the second order expression for the focal plane position are

$$x_f = \mathbf{T}_2[(x, \theta_x, y, \theta_y, \delta_E)_i], \quad (\text{A4})$$

$$= \mathbf{T}_1 + (x|xx)x_i^2 + (x|\theta_x\theta_x)\theta_i^2 + (x|\delta_E\delta_E)\delta_{Ei}^2, \quad (\text{A5})$$

$$+ (x|x\theta_x)x_i\theta_{xi} + (x|x\delta_E)x_i\delta_{Ei} + (x|\theta_x\delta_E)\theta_{xi}\delta_{Ei}, \quad (\text{A6})$$

$$+ (x|yy)y_i^2 + (x|xy)x_iy_i + (x|y\theta_y)y_i\theta_{yi} + \dots \quad (\text{A7})$$

The higher order terms produce aberrations. Geometric aberrations are proportional to the initial displacement and angle of particles and increase the effective magnification, thereby degrading the resolution. Chromatic aberrations are proportional to the energy

deviation and manifest as distortions of the focal plane, hampering integration with detectors. The focal plane rotation is a second order chromatic aberration,¹⁵

$$\tan \phi_{fp} = \frac{(x|\theta_x\delta_E)}{(\theta_x|\theta_x)(x|\delta_E)}. \quad (\text{A8})$$

The curvature of the focal plane is a third order chromatic aberration. The radius of curvature of a spectrometer with no focal plane rotation is

$$R_C = \frac{(x|\delta_E)^2}{(x|x)(x|\theta_x\delta_E\delta_E)}. \quad (\text{A9})$$

The central energy resolution, $\delta E/E$, of a spectrometer can be computed from a nonlinear transfer map by dividing the Full Width at Half Maximum (FWHM) of a monoenergetic image by the linear dispersion,

$$R[\%] = \frac{\text{FWHM}}{(x|\delta_E)}. \quad (\text{A10})$$

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