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Food security under trade policies and perishability: Extensions of the multicommodity network equilibrium model with monotonicity analysis

Francesco Biancalani [a], Giorgio Gnecco [a], Mauro Passacantando [b,*], Fabio Raciti [c]

[a] AXES Research Unit, IMT School for Advanced Studies, Piazza S. Francesco 19, 55100, Lucca, Italy
[b] Department of Business and Law, University of Milano-Bicocca, Via degli Arcimboldi 8, 20126, Milan, Italy
[c] Department of Mathematics and Computer Science, University of Catania, Viale A. Doria 6, 95125, Catania, Italy

HIGHLIGHTS

- We develop an international trade equilibrium model that explicitly incorporates tariffs, nutritional constraints, and product perishability.
- The model is formulated as a variational inequality problem.
- We provide a wide class of non-separable price and cost functions that guarantee the strong monotonicity of the variational inequality operator.
- We conduct numerical experiments using a real-world dataset on wheat trade.

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ABSTRACT

Building on recent work about multicommodity agricultural trade and food security, this paper develops a network equilibrium model with nutritional constraints formulated as a variational inequality problem. A key contribution of the paper is its monotonicity analysis of the associated operator. We establish sufficient conditions for strong monotonicity under general and separable affine price and transportation cost functions, thereby providing existence and uniqueness guarantees for equilibrium solutions. The framework is further extended to incorporate *ad valorem* tariffs and commodity perishability, allowing the analysis of more realistic international trade settings. Numerical experiments based on global wheat trade data illustrate the effects of nutritional requirements, trade policies, and perishability on equilibrium trade patterns and food security outcomes.

1. Introduction

Food insecurity remains one of the most urgent global challenges of our time. According to the World Food Program (WFP) [1], an estimated 343 million people are acutely food insecure across the 74 countries where WFP has an operational presence. This is nearly 200 million more than before the pandemic, reflecting the compounded impacts of COVID-19, conflict, climate change, and economic

* Corresponding author.
Email addresses: francesco.biancalani@alumni.imtlucca.it (F. Biancalani), giorgio.gnecco@imtlucca.it (G. Gnecco), mauro.passacantando@unimib.it (M. Passacantando), fabio.raciti@unict.it (F. Raciti).

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instability. The scale of the crisis is stark: 44.4 million people need urgent life-saving assistance, and up to 1.9 million are on the brink of famine. Catastrophic hunger has been reported in Gaza and Sudan, with acute need in South Sudan, Haiti, and Mali. Famine was confirmed in 2024 in the Zamzam camp in northern Sudan, where hundreds of thousands of displaced people are sheltered. Conflict is the primary driver of food insecurity, with 65 percent of people living in acutely food-insecure settings in 2024 being located in fragile or conflict-affected areas, and it also contributes to the increase in migratory flows [2]. Rising debt, high interest rates, and persistent food inflation have further eroded household purchasing power, with food prices more than doubling in 26 countries over the past four years. Extreme weather has also played a significant role: the 2023–24 El Niño brought severe droughts to Southern Africa, reducing maize production by 50 percent in Zimbabwe and Zambia, and affecting over 30 million people. Meanwhile, devastating floods across the Sahel, South Sudan, and parts of Asia impacted 12 million people. Despite these challenges, WFP emphasizes that hunger and malnutrition are solvable problems, combining immediate life-saving aid with long-term resilience-building initiatives.

In this context, mathematical models of international trade are crucial for capturing the complex interactions among supply, demand, prices, and policies across multiple countries and commodities. They provide a rigorous framework for evaluating how tariffs, subsidies, and resource constraints impact both trade flows and food security outcomes, thereby offering policymakers valuable insights.

This work extends the spatial price equilibrium model in [3], which builds upon the foundational framework of Samuelson [4] and Takayama and Judge [5,6], by introducing nutritional constraints akin to the classic diet problem in Operations Research [7,8]. We formulate the problem leveraging the variational inequality framework of Dafermos and Nagurney [9,10].

1.1. Related works

The foundational starting point of our analysis is the framework established by Nagurney [3], which integrates nutritional policies into international trade by extending the classic spatial price equilibrium model via the theory of variational inequalities. This approach represents a novel synthesis of international trade, multicommodity network equilibrium, and minimum nutritional standards coupled with government interventions. Structurally, the model characterizes spatially separated supply and demand markets governed by commodity-specific price functions and unit transportation costs. The resulting equilibrium conditions encompass both multicommodity trade flows and Lagrange multipliers, which serve as shadow prices for minimum nutrient standards.

Formulated as an alternative variational inequality, this framework allows for a rigorous analysis of existence and uniqueness properties, while establishing an efficient solution algorithm. Crucially, it provides explicit formulas to quantify the consumer subsidies required to meet nutritional baselines and safeguard food security. While Nagurney [3] demonstrated the practical utility of this model through a three-country case study (Ukraine, Egypt, and Lebanon) to illustrate how localized disruptions propagate across borders, contemporary literature has expanded upon this network architecture along two primary dimensions: product perishability and macroeconomic interventions (such as variations in tariffs).

Regarding product perishability, various methodologies have been employed to model cargo decay during transit. For instance, Nagurney [11] and Yu and Nagurney [12] captured temporal food decay by introducing fixed arc multipliers determined by transit duration and environmental conditions, with the latter work also accounting for waste disposal costs across supply chain stages.

To track quality more precisely, Nagurney et al. [13] integrated explicit formulas that combine time, temperature, and link capacities with upper limits on the quality of the initial product at the harvest sites. More recently, Nagurney and Besik [14] advanced this methodology by developing flow-dependent arc multipliers, where the fraction of lost cargo scales dynamically with traffic volume to capture real-world logistics bottlenecks and congestion.

In parallel, the representation of trade interventions within the variational inequality framework has evolved to encompass broader macroeconomic factors and structural shocks. Nagurney et al. [15] extended these equilibrium models by evaluating the simultaneous impacts of exchange rate fluctuations and trade policy instruments. Subsequently, the same authors expanded this multicommodity framework to assess international agricultural trade performance under catastrophic supply chain and capacity disruptions [16,17].

Despite these advancements, notable limitations persist in the existing literature. First, prior models generally treat tariffs as fixed unit costs (i.e., a constant monetary amount per ton of commodity), as in Nagurney [11] and in Nagurney et al. [15]. While mathematically tractable, this linear formulation fails to capture the proportional price distortions typical of modern *ad valorem* international trade policies. Furthermore, no existing model integrates network perishability and proportional tariffs alongside explicit human nutritional requirements.

1.2. Original contributions

Our model bridges critical gaps in the literature by uniquely integrating previously isolated features and fundamentally redefining core modeling assumptions. While the problem is formulated as a variational inequality drawing on the mathematical foundations established by Nagurney [3], our framework significantly expands both the scope and complexity of prior models. Specifically, we scale the network architecture to a global scope encompassing more than 40 countries (or groups of countries), in which nations face distinct nutritional constraints driven by exogenous factors (e.g., cultural aspects). Within this framework, we rigorously evaluate systemic resilience under a suite of highly differentiated disruption scenarios, as detailed in the numerical results.

The core contribution of this paper lies in advancing both the conceptual realism and the mathematical rigor of spatial network modeling. First, we embed shipment perishability – modeled as distance-dependent physical losses in transit – directly into a network governed by nutritional demands. This novel integration ensures that the computed economic equilibrium successfully satisfies human dietary requirements despite the continuous loss of cargo mass.[1] Furthermore, we depart from the fixed unit tariffs traditional in the literature [11,15] by explicitly modeling *ad valorem* tariffs as a percentage of the combined supply price and transportation cost. This proportional structure better reflects actual international trade policies. Additionally, to underpin these structural advancements, we prove sufficient conditions for the strong monotonicity of the variational inequality map. In particular, we provide classes of (general and separable) affine price and cost functions that guarantee strong monotonicity. Establishing these analytical conditions represents a critical technical milestone, as such rigorous mathematical foundations have remained unaddressed in related spatial network studies displaying similar structural features (see, e.g., [11,14–17]). Finally, our numerical experiments are based on real-world data, which we also use to rigorously estimate key parameters via a dedicated calibration procedure.

1.3. Organization of the work

The paper is structured as follows. Section 2 introduces the generalized international trade network equilibrium model with tariffs, nutritional constraints, and perishability. Section 3 provides the monotonicity analysis of the corresponding variational inequality operator, establishing sufficient conditions for strong monotonicity under both general and separable affine specifications of price and cost functions. Section 4 presents numerical experiments based on real-world data for the global wheat trade, illustrating the effects of tariffs and perishability under several policy scenarios. Finally, Section 5 concludes the paper and outlines directions for future research.

Throughout the paper, we use the symbol $[n]$ to denote the set $\{1,\dots,n\}$.

2. The generalized international trade network equilibrium model

In our model, we consider n country/group of countries supply markets and n country/group of countries demand markets. The supply markets and demand markets are engaged in the production and consumption, respectively, of H food commodities, with a typical commodity denoted by h. A representative supply market is indexed by i, and a representative demand market by j.

We denote by s_i^h the supply of commodity $h \in [H]$, in the supply market of country/group of countries $i \in [n]$. All supplies are collected in the column vector $s \in \mathbb{R}_+^{Hn}$. Similarly, d_j^h denotes the demand for commodity $h \in [H]$, in the demand market of country/group of countries $j \in [n]$. All demands are collected in the column vector $d \in \mathbb{R}_+^{Hn}$. The shipment of commodity $h \in [H]$ from the supply market of country/group of countries $i \in [n]$ to the demand market of country/group of countries $j \in [n]$ is denoted by Q_{ij}^h. All shipments are collected in the column vector $Q \in \mathbb{R}_+^{Hn^2}$.

Let e_i denote the exchange rate from the currency of country/group of countries i to the US dollar. Moreover, τ_{ij}^h denotes the tariff that country/group of countries j imposes on commodity h imported from country/group of countries i, with $i \neq j$. For simplicity, when countries are aggregated in a group, we assume that τ_{ij}^h is uniform over all such countries (a similar remark holds for other quantities introduced later). Within a generic country/group of countries i, let $\tau_{ii}^h = 0$.

Remark 1. Tariffs are typically modeled as additive duties or proportional *ad valorem* taxes based on either Free On Board (FOB) or Cost, Insurance, and Freight (CIF) valuations, all of which are fully permitted under World Trade Organization (WTO) frameworks. While the FOB model considers only the product's origin value, the CIF model further includes international transportation and insurance costs. This paper adopts the CIF framework, assuming that proportional tariffs are legally imposed on and paid by the importer in the destination country based on the total initial CIF value of the shipment.[2] This assumption aligns closely with standard global practices, since CIF is the most commonly used tariff valuation model globally, utilized by the European Union and the vast majority of international trading partners. Although the CIF framework nominally includes insurance, we do not model insurance costs explicitly in the paper, as they are generally negligible relative to freight costs and can be implicitly subsumed within the broader class of transaction costs. While the vast majority of real-world tariffs today are *ad valorem*, previous literature on variational inequalities for international trade has largely relied on unit tariffs for analytical simplicity [11,15]. Accounting for *ad valorem* tariffs therefore makes our modeling framework highly realistic.

A typical nutrient is denoted by l, with a total of L nutrients. We require that the population in the country/group of countries demand market j receive, for each l, at least a minimum quantity t_j^l. A representative consumer is assumed for the nutritional requirements. Furthermore, α^{lh} denotes the amount of nutrient $l \in [L]$ contained in one unit of commodity $h \in [H]$.

Finally, we allow for the possibility that a fraction of any commodity shipped spoils during transport and denote by r_{ij}^h the fraction of commodity h that arrives from i to j in good condition. The parameters and variables are shown in Tables 1 and 2, respectively.

We now introduce the supply and demand price functions. Moreover, to describe possible congestion effects and competition among markets, the unit transportation cost is considered a function of the entire vector of shipments:

$\bar{\pi}_i^h(s)$ is the supply price function for commodity $h \in [H]$ in the supply market of country/group of countries $i \in [n]$. All supply price functions are collected in the vector $\bar{\pi}(s) \in \mathbb{R}^{Hn}$.

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[1] For a foundational perspective outside of network modeling, the interested reader can also refer to the seminal general equilibrium trade model by Krugman [18], which utilizes the famous "iceberg" transport cost mechanism. Under this conceptualization, a fraction of the shipped goods metaphorically "melts away" before reaching their destination, meaning the cost of transit is inherently captured by the physical loss of the goods themselves (a dynamic that conceptually parallels the arc multipliers in our spatial network).

[2] Consequently, the importer incurs tariffs on the entire shipped volume, including the portion that degrades or spoils due to transit perishability.

Table 1
Model parameters.

Parameter	Description
τ_{ij}^h	Tariff that country/group of countries j imposes on commodity h imported from country/group of countries i.
t_j^l	Minimum required amount of nutrient l for the population in the demand market of country/group of countries j.
α^{lh}	Amount of nutrient l contained in one unit of commodity h.
r_{ij}^h	Fraction of the product h that arrives from i to j after spoilage.
e_i	Exchange rate from the currency of country/group of countries i to the US dollar.

Table 2
Model variables.

Variable	Description
s_i^h	Supply of commodity h in the supply market of country/group of countries i, with $s \in \mathbb{R}_+^{Hn}$.
d_j^h	Demand for commodity h in the demand market of country/group of countries j, with $d \in \mathbb{R}_+^{Hn}$.
Q_{ij}^h	Shipment of commodity h from the supply market of country/group of countries i to the demand market of country/group of countries j, with $Q \in \mathbb{R}_+^{Hn^2}$.

$\bar{\rho}_j^h(d)$ is the demand price function for commodity $h \in [H]$ in the demand market of country/group of countries $j \in [n]$. All demand price functions are collected in the vector $\bar{\rho}(d) \in \mathbb{R}^{Hn}$.

$c_{ij}^h(Q)$ is the unit transportation cost associated with the shipment of commodity $h \in [H]$ from the supply market of country/group of countries $i \in [n]$ to the demand market of country/group of countries $j \in [n]$. All transportation costs are collected in the vector $c(Q) \in \mathbb{R}_+^{Hn^2}$.

The two price functions above will later be transformed using the following flow conservation equations:

$$s_i^h = \sum_{j\in[n]} Q_{ij}^h, \qquad h \in [H],\ i \in [n], \tag{1}$$

$$d_j^h = \sum_{i\in[n]} r_{ij}^h Q_{ij}^h, \qquad h \in [H],\ j \in [n]. \tag{2}$$

Eq. (1) states that the supply of commodity h in country/group of countries i is equal to the total shipments of that commodity from i to all demand markets. Eq. (2) states that the demand for commodity h in country/group of countries j is equal to the total effective shipments of that commodity received from all supply markets, taking into account the spoilage factor r_{ij}^h. Moreover, we require that the flows of commodities cannot be negative:

$$Q_{ij}^h \geq 0, \qquad h \in [H],\ i \in [n],\ j \in [n]. \tag{3}$$

Nutritional constraints for food security. The nutritional constraints for food security ensure that the nutritional requirements of the population in each country/group of countries are satisfied. They are given by

$$\sum_{h\in[H]} \alpha^{lh} d_j^h \geq t_j^l, \qquad l \in [L],\ j \in [n]. \tag{4}$$

Constraints (4) state that the total intake of nutrient l in the demand market of country/group of countries j, obtained from all commodities consumed there, must be at least equal to the required minimum level t_j^l. In view of equations (2), constraints (4) can be rewritten as

$$\sum_{h\in[H]} \sum_{i\in[n]} \alpha^{lh} r_{ij}^h Q_{ij}^h \geq t_j^l, \qquad l \in [L],\ j \in [n], \tag{5}$$

and express that the effective amount of nutrient l arriving in country/group of countries j, after accounting for the spoilage factor r_{ij}^h, must meet or exceed the nutritional standard t_j^l. In the formulation of the international trade equilibrium conditions, we associate a Lagrange multiplier $\lambda_j^l \geq 0$ with constraint (5), corresponding to nutrient l and the demand market of country/group of countries j.

New multicommodity supply price functions $\pi_i^h(Q)$ for $h \in [H]$, $i \in [n]$, and new multicommodity demand price functions $\rho_j^h(Q)$ for $h \in [H]$, $j \in [n]$, which depend on the shipments of commodities, can now be defined as follows:

$$\pi_i^h \equiv \pi_i^h(Q) = \bar{\pi}_i^h(s), \qquad h \in [H],\ i \in [n], \tag{6}$$

and

$$\rho_j^h \equiv \rho_j^h(Q) = \bar{\rho}_j^h(d), \qquad h \in [H],\ j \in [n]. \tag{7}$$

In order to present the equilibrium conditions, we introduce the set of feasible shipments

$$\mathcal{K} = \left\{ Q \in \mathbb{R}_+^{Hn^2} \; : \; \sum_{h\in[H]} \sum_{i\in[n]} \alpha^{lh} r_{ij}^h Q_{ij}^h \geq t_j^l, \quad l \in [L],\; j \in [n] \right\}.$$

Definition 1 (Equilibrium in terms of a variational inequality). A multicommodity shipment flow $Q^* \in \mathcal{K}$ is an equilibrium of international trade with minimum nutritional standards for food security if it is a solution to the following variational inequality problem:

$$\sum_{h\in[H]} \sum_{i\in[n]} \sum_{j\in[n]} \left\{ \left[\pi_i^h(Q^*) + c_{ij}^h(Q^*) \right] e_i \left(1 + \tau_{ij}^h \right) - \rho_j^h(Q^*) r_{ij}^h e_j \right\} \left(Q_{ij}^h - Q_{ij}^{h*} \right) \geq 0, \qquad \forall\, Q \in \mathcal{K}. \tag{8}$$

Mathematically, the variational inequality (8) acts as an aggregate condition, stipulating that replacing the equilibrium flow Q^* with any feasible alternative flow $Q \in \mathcal{K}$ cannot yield a negative aggregate net deviation. Crucially, the prices and transaction costs within the brackets depend strictly on the equilibrium flow Q^* rather than the hypothetical alternative flow Q. Economically, this structure precisely captures a competitive market populated by infinitesimal (nonatomic) players. Because these trading agents act as strict price-takers with no individual bargaining power to alter, e.g., global prices or tariff structures, they evaluate alternative shipment strategies based entirely on the prevailing market realities locked in at Q^*. Consequently, while (8) is expressed as a global formulation, it is fundamentally the aggregate of these individual choices, ensuring that no single agent can unilaterally improve his/her outcome. The formal equivalence between this aggregate variational inequality and suitable micro-level equilibrium conditions is established in the following theorem.

Theorem 1 (Equivalent equilibrium conditions). *A multicommodity shipment flow $Q^* \in \mathcal{K}$ solves the variational inequality problem (8) if and only if it satisfies the following conditions: for each commodity $h \in [H]$, for each nutrient $l \in [L]$ and for each pair of country/group of countries' supply and demand markets $(i,j) \in [n] \times [n]$:*

$$\left[\pi_i^h(Q^*) + c_{ij}^h(Q^*) \right] e_i \left(1 + \tau_{ij}^h \right) \begin{cases} = \rho_j^h(Q^*)\, r_{ij}^h\, e_j + \sum\limits_{l\in[L]} \alpha^{lh}\, r_{ij}^h \lambda_j^{l*}, & \text{if } Q_{ij}^{h*} > 0, \\ \geq \rho_j^h(Q^*)\, r_{ij}^h\, e_j + \sum\limits_{l\in[L]} \alpha^{lh}\, r_{ij}^h \lambda_j^{l*}, & \text{if } Q_{ij}^{h*} = 0, \end{cases} \tag{9}$$

$$\sum_{h\in[H]} \sum_{i\in[n]} \alpha^{lh} r_{ij}^h Q_{ij}^{h*} - t_j^l \begin{cases} = 0, & \text{if } \lambda_j^{l*} > 0, \\ \geq 0, & \text{if } \lambda_j^{l*} = 0, \end{cases} \tag{10}$$

where $\lambda^* \in \mathbb{R}_+^{Ln}$ is the optimal Lagrange multiplier vector associated with constraints (5).

Proof. We will prove the equivalence by writing the KKT conditions associated with the variational inequality (8). For this, consider the function

$$\phi(Q) = \sum_{h\in[H]} \sum_{i\in[n]} \sum_{j\in[n]} \left\{ \left[\pi_i^h(Q^*) + c_{ij}^h(Q^*) \right] e_i \left(1 + \tau_{ij}^h \right) - \rho_j^h(Q^*) r_{ij}^h e_j \right\} Q_{ij}^h, \tag{11}$$

and note that Q^* is a solution of (8) if and only if Q^* is a global minimum point for ϕ over $\mathcal{K}$. Let us denote with λ_j^l, with $j \in [n]$, $l \in [L]$, the Lagrange multipliers associated with the nutritional constraints in $\mathcal{K}$. Since all the components of Q are nonnegative, we write the Lagrangian with respect to the nutritional constraints only:

$$\mathcal{L}(Q,\lambda) = \phi(Q) + \sum_{j\in[n]} \sum_{l\in[L]} \lambda_j^l \left(t_j^l - \sum_{h\in[H]} \sum_{i\in[n]} \alpha^{lh}\, r_{ij}^h Q_{ij}^h \right)$$

and the following KKT conditions:

$$\frac{\partial \mathcal{L}(Q^*,\lambda^*)}{\partial Q_{ij}^h} \geq 0,\; Q_{ij}^{h*} \geq 0,\; \frac{\partial \mathcal{L}(Q^*,\lambda^*)}{\partial Q_{ij}^h}\, Q_{ij}^{h*} = 0, \qquad \forall\, h \in [H],\; i,j \in [n], \tag{12}$$

$$\lambda_j^{l*} \left(t_j^l - \sum_{h\in[H]} \sum_{i\in[n]} \alpha^{lh}\, r_{ij}^h Q_{ij}^{h*} \right) = 0, \qquad \forall\, l \in [L],\; j \in [n], \tag{13}$$

$$\lambda_j^{l*} \geq 0, \qquad t_j^l - \sum_{h\in[H]} \sum_{i\in[n]} \alpha^{lh}\, r_{ij}^h Q_{ij}^{h*} \leq 0, \qquad \forall\, l \in [L],\; j \in [n]. \tag{14}$$

After some algebra we get that (12)–(14) coincide with (9)–(10), and the proof is completed. □

Remark 2. It is worth remarking that, while real-world international agricultural markets face complexities due to spatial separation, currency conversions, and product spoilage, the formulation (8) maintains the assumption of price-taking competition both domestically and internationally. Indeed, the physical perishability of goods during transit (captured by the survival fraction r_{ij}^h), the unit transportation costs ($c_{ij}^h(Q^*)$), and the application of exchange rates (e_i and e_j) do not introduce market power or restrict price-taking behavior. Instead, they function collectively as physical, spatial, and financial frictions. Trading agents remain strictly competitive, infinitesimal price-takers who must simply internalize transit losses, freight rates, and currency valuations into their spatial arbitrage decisions. In other words, these factors alter the optimal trade volumes but do not violate the underlying price-taking competition framework of the model.

Remark 3. Building on the well-known isomorphism between spatial price and traffic network equilibria [19], the equilibrium conditions (9) and (10) extend Wardrop's First Principle – also known as user equilibrium, where no infinitesimal driver can lower his/her travel cost by unilaterally switching routes – to international trade where all the infinitesimal producers/consumers are price-takers. For active links ($Q_{ij}^{h*} > 0$), the exchange-rate and tariff-adjusted supply and transaction costs incurred by sellers in country/group of countries i exactly balance the destination price and food security premium (λ_j^{l*}) faced by buyers in country/group of countries j, both scaled by the product fraction surviving spoilage (r_{ij}^h). Unused links ($Q_{ij}^{h*} = 0$) face costs larger than or equal to this spoilage-adjusted payoff. Just as drivers minimize travel time, trade flows adjust at the equilibrium until no seller can find a more profitable connection.

In the equilibrium condition (9), the term $\sum_{l\in[L]} \alpha^{lh} r_{ij}^h \lambda_j^{l*}$ for each commodity h and demand market j can be interpreted as an additional "value" associated with the commodity h in the demand market of country/group of countries j. In fact, it represents the extra price per unit of that commodity that ensures shipments are sufficient to satisfy the minimum nutritional standards in the country/group of countries, taking into account the nutrient content and perishability. Without such a subsidy for each commodity h, the volume of shipments to its demand market j might not be enough to meet the nutritional standards (also taking its perishability into account), since consumers would be willing to pay only $\rho_j^h(Q^*) r_{ij}^h$ for the commodity h at j in the absence of the subsidy (again, taking into account its perishability).

Moreover, this consumption subsidy is defined per "unit" of the commodity, which in the case of agricultural goods in international trade, typically corresponds to one metric ton. The total financial outlay by the government(s) of country/group of countries j for commodity h is therefore

$$\sum_{l\in[L]} \sum_{i\in[n]} \alpha^{lh}\, r_{ij}^h\, \lambda_j^{l*}\, Q_{ij}^{h*},$$

with the total expenditure covering subsidies for all commodities given by

$$\sum_{h\in[H]} \sum_{l\in[L]} \sum_{i\in[n]} \alpha^{lh}\, r_{ij}^h\, \lambda_j^{l*} Q_{ij}^{h*}.$$

The variational inequality problem (8) can be written in compact form as follows:

$$\text{find } Q^* \in \mathcal{K} \text{ such that } F(Q^*)^\top (Q - Q^*) \geq 0, \quad \forall\, Q \in \mathcal{K}, \tag{$VI(F,\mathcal{K})$}$$

where $F : \mathbb{R}^{Hn^2} \to \mathbb{R}^{Hn^2}$ is the map such that $F(Q)$ is the vector with components $\left([\pi_i^h(Q) + c_{ij}^h(Q)] e_i (1 + \tau_{ij}^h) - \rho_j^h(Q) r_{ij}^h e_j \right)_{hij}$ ordered in the same manner as Q_{ij}^h. For an account of variational inequalities, the interested reader can refer to [20,21]. We recall here some useful monotonicity properties.

Definition 2. An operator $A : \mathbb{R}^n \to \mathbb{R}^n$ is said to be monotone on a set $K \subset \mathbb{R}^n$ iff

$$[A(x) - A(y)]^\top (x - y) \geq 0, \qquad \forall\, x, y \in K.$$

A is said strongly monotone on K iff there exists $\eta > 0$ such that

$$[A(x) - A(y)]^\top (x - y) \geq \eta \|x - y\|^2, \qquad \forall\, x, y \in K.$$

Conditions that ensure the unique solvability of a variational inequality problem are given by the following theorem (see, e.g., [10, 20]).

Theorem 2. *If the set $K \subset \mathbb{R}^n$ is nonempty, closed and convex and the map $A : \mathbb{R}^n \to \mathbb{R}^n$ is continuous and strongly monotone on K, then the variational inequality problem $VI(A, K)$ admits a unique solution.*

3. Monotonicity analysis

In this section, we first prove that a wide class of general affine price and cost functions guarantees the strong monotonicity of the map F. Next, we present a subclass of separable affine functions that ensures this property. A similar analysis has been carried out for a different model in [22].

Theorem 3. *Assume that any supply price function is affine, i.e.,*

$$\bar{\pi}_i^h(s) = a_i^h + \sum_{i'\in[n]} \sum_{h'\in[H]} \alpha_{ii'}^{hh'} s_{i'}^{h'}, \tag{15}$$

with $\alpha_{ii'}^{hh'} > 0$, for any $h, h' \in [H]$, $i, i' \in [n]$; any transportation cost function is affine, i.e.,

$$c_{ij}^h(Q) = b_{ij}^h + \sum_{i'\in[n]} \sum_{j'\in[n]} \sum_{h'\in[H]} \beta_{iji'j'}^{hh'} Q_{i'j'}^{h'}, \tag{16}$$

with $\beta^{hh'}_{iji'j'} > 0$, for any $h, h' \in [H]$, $i, j, i', j' \in [n]$, and any demand price function is affine, i.e.,

$$\bar{\rho}^h_j(d) = g^h_j - \sum_{j' \in [n]} \sum_{h' \in [H]} \gamma^{hh'}_{jj'} d^{h'}_{j'}, \tag{17}$$

with $\gamma^{hh'}_{jj'} > 0$, for any $h, h' \in [H]$, $j, j' \in [n]$. If the following condition holds:

$$\begin{aligned}
&\min_{\substack{h\in[H]\\ i,j\in[n]}} \left\{ e_i \left(1+\tau^h_{ij}\right) \alpha^{hh}_{ii} - \sum_{(h',i',j')\neq(h,i,j)} \frac{e_i\left(1+\tau^h_{ij}\right)\alpha^{hh'}_{ii'} + e_{i'}\left(1+\tau^{h'}_{i'j'}\right)\alpha^{h'h}_{i'i}}{2} \right\} \\
&+ \min_{\substack{h\in[H]\\ i,j\in[n]}} \left\{ e_i \left(1+\tau^h_{ij}\right) \beta^{hh}_{ijij} - \sum_{(h',i',j')\neq(h,i,j)} \frac{e_i\left(1+\tau^h_{ij}\right)\beta^{hh'}_{iji'j'} + e_{i'}\left(1+\tau^{h'}_{i'j'}\right)\beta^{h'h}_{i'j'ij}}{2} \right\} \\
&+ \min_{\substack{h\in[H]\\ i,j\in[n]}} \left\{ e_j \left(r^h_{ij}\right)^2 \gamma^{hh}_{jj} - \sum_{(h',i',j')\neq(h,i,j)} \frac{r^h_{ij} e_j r^{h'}_{i'j'} \gamma^{hh'}_{jj'} + r^{h'}_{i'j'} e_{j'} r^h_{ij} \gamma^{h'h}_{j'j}}{2} \right\} > 0,
\end{aligned} \tag{18}$$

then F is strongly monotone on $\mathbb{R}^{Hn^2}$.

Proof. The map F can be decomposed as the sum of three maps: $F = F_1 + F_2 + F_3$, where $(F_1(Q))_{hij} = \pi^h_i(Q) e_i (1+\tau^h_{ij})$, $(F_2(Q))_{hij} = c^h_{ij}(Q) e_i (1+\tau^h_{ij})$, and $(F_3(Q))_{hij} = -\rho^h_j(Q) r^h_{ij} e_j$.

Assumption (15) implies that F_1 is an affine map with

$$\left(F_1(Q)\right)_{hij} = e_i\left(1+\tau^h_{ij}\right)\left[a^h_i + \sum_{i'\in[n]} \sum_{h'\in[H]} \sum_{j'\in[n]} \alpha^{hh'}_{ii'} Q^{h'}_{i'j'}\right],$$

thus the Jacobian matrix of F_1 is constant with

$$\left(\nabla F_1\right)_{hij,h'i'j'} = e_i\left(1+\tau^h_{ij}\right)\alpha^{hh'}_{ii'},$$

and its symmetric part is

$$\left(\frac{\nabla F_1 + \nabla F_1^\top}{2}\right)_{hij,h'i'j'} = \frac{e_i\left(1+\tau^h_{ij}\right)\alpha^{hh'}_{ii'} + e_{i'}\left(1+\tau^{h'}_{i'j'}\right)\alpha^{h'h}_{i'i}}{2}.$$

The Gershgorin theorem guarantees that the following lower bound on the minimum eigenvalue of $(\nabla F_1 + \nabla F_1^\top)/2$ holds:

$$\lambda_{\min}\left(\frac{\nabla F_1 + \nabla F_1^\top}{2}\right) \geq \min_{\substack{h\in[H]\\ i,j\in[n]}} \left\{ e_i \left(1+\tau^h_{ij}\right) \alpha^{hh}_{ii} - \sum_{(h',i',j')\neq(h,i,j)} \frac{e_i\left(1+\tau^h_{ij}\right)\alpha^{hh'}_{ii'} + e_{i'}\left(1+\tau^{h'}_{i'j'}\right)\alpha^{h'h}_{i'i}}{2} \right\} := \mu_1.$$

Assumption (16) implies that F_2 is an affine map with

$$\left(F_2(Q)\right)_{hij} = e_i\left(1+\tau^h_{ij}\right)\left[b^h_{ij} + \sum_{i'\in[n]} \sum_{j'\in[n]} \sum_{h'\in[H]} \beta^{hh'}_{iji'j'} Q^{h'}_{i'j'}\right],$$

thus the Jacobian matrix of F_2 is constant with

$$\left(\nabla F_2\right)_{hij,h'i'j'} = e_i\left(1+\tau^h_{ij}\right)\beta^{hh'}_{iji'j'},$$

and its symmetric part is

$$\left(\frac{\nabla F_2 + \nabla F_2^\top}{2}\right)_{hij,h'i'j'} = \frac{e_i\left(1+\tau^h_{ij}\right)\beta^{hh'}_{iji'j'} + e_{i'}\left(1+\tau^{h'}_{i'j'}\right)\beta^{h'h}_{i'j'ij}}{2}.$$

The Gershgorin theorem implies that

$$\lambda_{\min}\left(\frac{\nabla F_2 + \nabla F_2^\top}{2}\right) \geq \min_{\substack{h\in[H]\\ i,j\in[n]}} \left\{ e_i \left(1+\tau^h_{ij}\right) \beta^{hh}_{ijij} - \sum_{(h',i',j')\neq(h,i,j)} \frac{e_i\left(1+\tau^h_{ij}\right)\beta^{hh'}_{iji'j'} + e_{i'}\left(1+\tau^{h'}_{i'j'}\right)\beta^{h'h}_{i'j'ij}}{2} \right\} := \mu_2.$$

Assumption (17) implies that F_3 is an affine map with

$$\left(F_3(Q)\right)_{hij} = r_{ij}^h e_j \left[\sum_{h' \in [H]} \sum_{i' \in [n]} \sum_{j' \in [n]} r_{i'j'}^{h'} \gamma_{jj'}^{hh'} Q_{i'j'}^{h'} - g_j^h \right],$$

thus the Jacobian matrix of F_3 is constant with

$$\left(\nabla F_3\right)_{hij,h'i'j'} = r_{ij}^h e_j r_{i'j'}^{h'} \gamma_{jj'}^{hh'},$$

and its symmetric part is

$$\left(\frac{\nabla F_3 + \nabla F_3^\top}{2}\right)_{hij,h'i'j'} = \frac{r_{ij}^h e_j r_{i'j'}^{h'} \gamma_{jj'}^{hh'} + r_{i'j'}^{h'} e_{j'} r_{ij}^h \gamma_{j'j}^{h'h}}{2}.$$

The Gershgorin theorem guarantees that the following lower bound on the minimum eigenvalue of $(\nabla F_3 + \nabla F_3^\top)/2$ holds:

$$\lambda_{\min}\left(\frac{\nabla F_3 + \nabla F_3^\top}{2}\right) \geq \min_{\substack{h \in [H] \\ i,j \in [n]}} \left\{ e_j \left(r_{ij}^h\right)^2 \gamma_{jj}^{hh} - \sum_{(h',i',j') \neq (h,i,j)} \frac{r_{ij}^h e_j r_{i'j'}^{h'} \gamma_{jj'}^{hh'} + r_{i'j'}^{h'} e_{j'} r_{ij}^h \gamma_{j'j}^{h'h}}{2} \right\} := \mu_3.$$

Given any $Q, Q' \in \mathbb{R}^{Hn^2}$, we have

$$\begin{aligned}
(Q - Q')^\top [(F_1 + F_2 + F_3)(Q) - (F_1 + F_2 + F_3)(Q')] &= (Q - Q')^\top \nabla F_1 (Q - Q') + (Q - Q')^\top \nabla F_2 (Q - Q') + (Q - Q')^\top \nabla F_3 (Q - Q') \\
&= (Q - Q')^\top \left(\left(\nabla F_1 + \nabla F_1^\top\right)/2\right)(Q - Q') + (Q - Q')^\top \left(\left(\nabla F_2 + \nabla F_2^\top\right)/2\right)(Q - Q') + (Q - Q')^\top \left(\left(\nabla F_3 + \nabla F_3^\top\right)/2\right)(Q - Q') \\
&\geq \left[\lambda_{\min}\left(\frac{\nabla F_1 + \nabla F_1^\top}{2}\right) + \lambda_{\min}\left(\frac{\nabla F_2 + \nabla F_2^\top}{2}\right) + \lambda_{\min}\left(\frac{\nabla F_3 + \nabla F_3^\top}{2}\right)\right] \|Q - Q'\|^2 \\
&\geq (\mu_1 + \mu_2 + \mu_3)\|Q - Q'\|^2.
\end{aligned}$$

Since condition (18) reads $\mu_1 + \mu_2 + \mu_3 > 0$, the map F is strongly monotone on $\mathbb{R}^{Hn^2}$. □

Theorem 3 deals with the most general case in which any price or cost function depends on all the possible variables. However, in applications, it is often interesting to consider the separable case in which the supply price function $\bar{\pi}_i^h$ (resp. demand price function $\bar{\rho}_j^h$) depends only on the supply of commodity h produced at market i (resp. the demand for commodity h in market j). The following result provides, in this particular case, a sufficient condition simpler than (18) for F to be strongly monotone.

Corollary 4. *Assume that any supply price function is affine and*

$$\bar{\pi}_i^h(s) = a_i^h + \alpha_{ii}^{hh} s_i^h, \tag{19}$$

with $\alpha_{ii}^{hh} > 0$ *for any* $h \in [H]$, $i \in [n]$, *any transportation cost function is affine and*

$$c_{ij}^h(Q) = b_{ij}^h + \beta_{ijij}^{hh} Q_{ij}^h, \tag{20}$$

with $\beta_{ijij}^{hh} > 0$ *for any* $h \in [H]$, $i, j \in [n]$, *any demand price function is affine and*

$$\bar{\rho}_j^h(d) = g_j^h - \gamma_{jj}^{hh} d_j^h, \tag{21}$$

with $\gamma_{jj}^{hh} > 0$ *for any* $j \in [n]$, $h \in [H]$. *If the following condition holds:*

$$\min_{\substack{h \in [H] \\ i \in [n]}} \frac{1}{2} \left\{ \sum_{j \in [n]} \alpha_{ii}^{hh} e_i \left(1 + \tau_{ij}^h\right) - \sqrt{n \sum_{j \in [n]} \left[\alpha_{ii}^{hh} e_i \left(1 + \tau_{ij}^h\right)\right]^2} \right\} + \min_{\substack{h \in [H] \\ i,j \in [n]}} \left\{ e_i \left(1 + \tau_{ij}^h\right) \beta_{ijij}^{hh} \right\} > 0, \tag{22}$$

then F *is strongly monotone on* $\mathbb{R}^{Hn^2}$.

Proof. As in the proof of Theorem 3, we decompose the map F as $F = F_1 + F_2 + F_3$, where $(F_1(Q))_{hij} = \pi_i^h(Q) e_i (1 + \tau_{ij}^h)$, $(F_2(Q))_{hij} = c_{ij}^h(Q) e_i (1 + \tau_{ij}^h)$, and $(F_3(Q))_{hij} = -\rho_j^h(Q) r_{ij}^h e_j$.

Assumption (19) implies that F_1 is an affine map with

$$\left(F_1(Q)\right)_{hij} = e_i \left(1 + \tau_{ij}^h\right) \left[a_i^h + \sum_{j' \in [n]} \alpha_{ii}^{hh} Q_{ij'}^h \right],$$

thus the Jacobian matrix of F_1 is

$$\left(\nabla F_1\right)_{hij,h'i'j'} = \begin{cases} e_i \left(1 + \tau_{ij}^h\right) \alpha_{ii}^{hh} & \text{if } (h', i') = (h, i), \\ 0 & \text{if } (h', i') \neq (h, i). \end{cases}$$

Hence, if we order the rows and columns of ∇F_1 according to the lexicographical order on the indices (h, i, j) and (h', i', j'), respectively, then ∇F_1 is a block diagonal matrix:

$$\nabla F_1 = \begin{pmatrix} B_1^1 & & & & & & 0 \\ & \ddots & & & & & \\ & & B_n^1 & & & & \\ & & & \ddots & & & \\ & & & & B_1^H & & \\ & & & & & \ddots & \\ 0 & & & & & & B_n^H \end{pmatrix}$$

where the generic block B_i^h, with $h \in [H]$, $i \in [n]$, is given by

$$B_i^h = \begin{pmatrix} \alpha_{ii}^{hh} e_i \left(1 + \tau_{i1}^h\right) & \dots & \alpha_{ii}^{hh} e_i \left(1 + \tau_{i1}^h\right) \\ \vdots & & \vdots \\ \alpha_{ii}^{hh} e_i \left(1 + \tau_{in}^h\right) & \dots & \alpha_{ii}^{hh} e_i \left(1 + \tau_{in}^h\right) \end{pmatrix}.$$

Notice that any block B_i^h is a rank-one matrix of the form

$$V = \begin{pmatrix} v_1 & \dots & v_1 \\ \vdots & & \vdots \\ v_n & \dots & v_n \end{pmatrix} = v\mathbf{1}^\top, \qquad \text{with} \quad v = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}, \mathbf{1} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}.$$

It is easy to check that the eigenvalues of $(V + V^\top)/2$ are

$$\lambda_1 = \left(v^\top \mathbf{1} - \sqrt{n}\|v\|\right)/2,$$
$$\lambda_2 = \left(v^\top \mathbf{1} + \sqrt{n}\|v\|\right)/2,$$
$$\lambda_3 = 0 \quad \text{with multiplicity} \quad n-2.$$

The Cauchy-Schwarz inequality guarantees that $\lambda_1 \le 0$, hence

$$\lambda_{\min}((V + V^\top)/2) = \left(v^\top \mathbf{1} - \sqrt{n}\|v\|\right)/2.$$

Therefore, we have

$$\lambda_{\min}\left(\frac{\nabla F_1 + \nabla F_1^\top}{2}\right) = \min_{\substack{h\in[H]\\ i\in[n]}} \lambda_{\min}\left(\frac{B_i^h + \left(B_i^h\right)^\top}{2}\right) = \min_{\substack{h\in[H]\\ i\in[n]}} \frac{1}{2}\left\{\sum_{j\in[n]} \alpha_{ii}^{hh} e_i \left(1 + \tau_{ij}^h\right) - \sqrt{n \sum_{j\in[n]} \left[\alpha_{ii}^{hh} e_i \left(1 + \tau_{ij}^h\right)\right]^2}\right\} := \mu_1. \tag{23}$$

On the other hand, assumption (20) implies that F_2 is an affine map with

$$\left(F_2(Q)\right)_{hij} = e_i \left(1 + \tau_{ij}^h\right)\left[b_{ij}^h + \beta_{ijij}^{hh} Q_{ij}^h\right],$$

thus the Jacobian matrix of F_2 is

$$\left(\nabla F_2\right)_{hij,h'i'j'} = \begin{cases} e_i \left(1 + \tau_{ij}^h\right)\beta_{ijij}^{hh} & \text{if } (h', i', j') = (h, i, j), \\ 0 & \text{if } (h', i', j') \neq (h, i, j), \end{cases}$$

that is a diagonal matrix with

$$\lambda_{\min}\left(\nabla F_2\right) = \min_{\substack{h\in[H]\\ i,j\in[n]}} \left\{e_i \left(1 + \tau_{ij}^h\right)\beta_{ijij}^{hh}\right\} := \mu_2. \tag{24}$$

Assumption (21) implies that F_3 is an affine map with

$$\left(F_3(Q)\right)_{hij} = r_{ij}^h e_j \gamma_{jj}^{hh} \sum_{i'\in[n]} r_{i'j}^h Q_{i'j}^h - r_{ij}^h e_j g_j^h,$$

thus the Jacobian matrix of F_3 is

$$\left(\nabla F_3\right)_{hij,h'i'j'} = \begin{cases} e_j \gamma_{jj}^{hh} r_{ij}^h r_{i'j}^h & \text{if } (h', j') = (h, j), \\ 0 & \text{if } (h', j') \neq (h, j). \end{cases}$$

Hence, if we order the rows and columns of ∇F_3 according to the lexicographical order on the indices (h, j, i) and (h', j', i'), respectively, then ∇F_3 is a block diagonal matrix:

$$\nabla F_3 = \begin{pmatrix} R_1^1 & & & & & & 0 \\ & \ddots & & & & & \\ & & R_n^1 & & & & \\ & & & \ddots & & & \\ & & & & R_1^H & & \\ & & & & & \ddots & \\ 0 & & & & & & R_n^H \end{pmatrix},$$

where the generic block R_j^h, with $h \in [H]$, $j \in [n]$, is given by

$$R_j^h = e_j \gamma_{jj}^{hh} \begin{pmatrix} r_{1j}^h r_{1j}^h & \cdots & r_{1j}^h r_{nj}^h \\ \vdots & & \vdots \\ r_{nj}^h r_{1j}^h & \cdots & r_{nj}^h r_{nj}^h \end{pmatrix} = e_j \gamma_{jj}^{hh} w_j^h \left(w_j^h \right)^\top, \quad \text{with} \quad w_j^h = \begin{pmatrix} r_{1j}^h \\ \vdots \\ r_{nj}^h \end{pmatrix}$$

Hence, any block R_j^h and the whole matrix ∇F_3 are positive semidefinite; thus F_3 is monotone on $\mathbb{R}^{Hn^2}$.

Finally, given any $Q, Q' \in \mathbb{R}^{Hn^2}$, we have

$$\begin{aligned}
&(Q - Q')^\top [(F_1 + F_2 + F_3)(Q) - (F_1 + F_2 + F_3)(Q')] \\
&\quad \geq (Q - Q')^\top [(F_1 + F_2)(Q) - (F_1 + F_2)(Q')] \\
&\quad = (Q - Q')^\top \nabla F_1 (Q - Q') + (Q - Q')^\top \nabla F_2 (Q - Q') \\
&\quad = (Q - Q')^\top \left(\left(\nabla F_1 + \nabla F_1^\top \right) / 2 \right) (Q - Q') + (Q - Q')^\top \nabla F_2 (Q - Q') \\
&\quad \geq \left[\lambda_{\min} \left(\frac{\nabla F_1 + \nabla F_1^\top}{2} \right) + \lambda_{\min} \left(\nabla F_2 \right) \right] \| Q - Q' \|^2 \\
&\quad = (\mu_1 + \mu_2) \| Q - Q' \|^2.
\end{aligned}$$

Since condition (22) reads $\mu_1 + \mu_2 > 0$, the map F is strongly monotone on $\mathbb{R}^{Hn^2}$. □

Remark 4. If the price and cost functions are separable as in (19)–(21) and there are no tariffs, i.e., $\tau_{ij}^h = 0$ for any $h \in [H]$, $i, j \in [n]$, then F is strongly monotone since condition (22) is satisfied. In fact, we have

$$\begin{aligned}
&\min_{\substack{h \in [H] \\ i \in [n]}} \frac{1}{2} \left\{ \sum_{j \in [n]} \alpha_{ii}^{hh} e_i \left(1 + \tau_{ij}^h \right) - \sqrt{n \sum_{j \in [n]} \left[\alpha_{ii}^{hh} e_i \left(1 + \tau_{ij}^h \right) \right]^2} \right\} + \min_{\substack{h \in [H] \\ i,j \in [n]}} \left\{ e_i \left(1 + \tau_{ij}^h \right) \beta_{ijij}^{hh} \right\} \\
&\quad = \min_{\substack{h \in [H] \\ i \in [n]}} \frac{1}{2} \left\{ n \alpha_{ii}^{hh} e_i - \sqrt{n^2 \left[\alpha_{ii}^{hh} e_i \right]^2} \right\} + \min_{\substack{h \in [H] \\ i,j \in [n]}} \left\{ e_i \beta_{ijij}^{hh} \right\} \\
&\quad = \min_{\substack{h \in [H] \\ i,j \in [n]}} \left\{ e_i \beta_{ijij}^{hh} \right\} > 0.
\end{aligned}$$

Remark 5. The case of routes involving multiple links is beyond the scope of the present paper and would require further analytical and numerical methods. Indeed, as shown in [23] for the traffic equilibrium problem, when routes consisting of more than one link are considered, the strict monotonicity of the underlying operator is generally lost, even if the link cost functions are strictly monotone. This is relevant in the context of the present work as any strongly monotone operator is strictly monotone, too. Analogously, allowing for a multigraph, with multiple links connecting the same origin–destination pair, would also affect our monotonicity analysis.

4. Numerical experiments

In this section, we present some numerical experiments using the equilibrium model described in Section 2, applied to a real-world dataset on wheat trade. In the following, we investigate six different scenarios. In all the considered scenarios, the supply price, demand price, and transportation cost functions are supposed to be in the form (19)–(21), so that the equilibrium model is equivalent to an affine variational inequality. The equilibrium shipment flow has been computed by reformulating the latter affine variational inequality as an equivalent quadratic optimization problem using the merit function approach proposed in [24]. The resulting quadratic program was solved using Gurobi 12.0.0 (with default options).

The basic scenario (Scenario 1) considers 42 countries (or groups of countries), one commodity (wheat), and the amount of energy supplied (kcalories) as nutritional constraint. The supply and demand price functions are supposed to be in the form (19) and (21), respectively, where the parameters have been obtained by following the first calibration procedure described in [25], based on the observed equilibrium prices and quantities,[3] and on elasticity values (mapped to slopes) taken from the literature. The resulting values of the parameters for the supply and demand price functions have been obtained directly from the code (see the file "`Replication Code.gms`") available as supplementary material of [25]. The transportation cost functions are assumed to be in the form (20), where the constant parameters b^h_{ij} are defined as in [25] and we set the coefficients $\beta^{hh}_{ijij} = 0.02$ as in [3]. The price and cost functions are in a common currency, the US dollar. The tariffs τ^h_{ij} between countries/group of countries are defined as in [25]. The spoilage factor r^h_{ij} is defined as a decreasing affine function of the parameter b^h_{ij} (which, based on the arguments reported in [25, Section 4], can be assumed to be a proxy of a variable directly proportional to the distance between i and j), i.e.,

$$r^h_{ij} = 1 - 0.1\frac{b^h_{ij}}{\max\limits_{i,j\in[n]} b^h_{ij}},$$

so that r^h_{ij} varies between 0.9 (corresponding to a pair of countries/group of countries at the greatest distance) and 1 (corresponding to the case where $i = j$ since $b^h_{ii} = 0$ holds for any $i \in [n]$). Here, we choose the value 0.1 for the constant as a structural compromise between the per-kilometer road transportation wheat losses estimated by [26] (see the next Footnote 5) and the fact that bulk rail and ocean freight do not suffer from continuous transit degradation. Unlike road networks – which experience uniquely high per-kilometer spillage and, alongside barges, account for 44% of inland wheat movement [27] – rail and maritime losses are strictly nodal (concentrated during terminal operations). Accommodating this difference is essential, as rail handles the majority (56%) of inland wheat movement [27] and maritime shipping facilitates virtually all global overseas wheat trade [28]. Indeed, air transport remains effectively at 0% commercial usage due to economic constraints – specifically, freight costs that vastly exceed the value of the commodity itself – rendering its theoretical transit loss irrelevant in the analysis.

Additionally, we set the parameter $\alpha^{lh} = 3,300,000$ kcalories per ton, as in [3]. As regards nutritional constraints, we assume a minimum of 500 kcal provided by wheat (and its derivatives) per day per capita for European countries, the USA, Canada, and Australia.[4]

Finally, to better compare the various scenarios, we introduce the Total Economic Surplus (TES) for any country/group of countries. This metric is defined by four distinct components associated with the specific country/group of countries: the sum of consumer surplus, producer surplus, and tariff revenues, minus government subsidy expenditures. For the case of affine and separable price supply and demand functions, the TES of each country (or group of countries) $j \in [n]$ is expressed as follows:

$$\begin{aligned}
TES_j &= \underbrace{e_j \sum_{h\in[H]} \left(\int_0^{d^{h*}_j} \bar{\rho}^h_j(x)dx - \bar{\rho}^h_j\left(d^{h*}_j\right) d^{h*}_j \right)}_{\text{Consumer Surplus}} + \underbrace{e_j \sum_{h\in[H]} \left(\bar{\pi}^h_j\left(s^{h*}_j\right) s^{h*}_j - \int_0^{s^{h*}_j} \bar{\pi}^h_j(y)dy \right)}_{\text{Producer Surplus}} \\
&\quad + \underbrace{\sum_{h\in[H]} \sum_{i\in[n]\setminus\{j\}} \tau^h_{ij} \left[\pi^h_i(Q^*) + c^h_{ij}(Q^*)\right] e_i Q^{h*}_{ij}}_{\text{Tariff Revenues}} - \underbrace{\sum_{l\in[L]} \sum_{h\in[H]} \sum_{i\in[n]} \alpha^{lh} r^h_{ij} \lambda^{l*}_j Q^{h*}_{ij}}_{\text{Government Subsidy Expenditure}} \\
&= \underbrace{e_j \sum_{h\in[H]} \frac{1}{2}\gamma^{hh}_{jj}\left(d^{h*}_j\right)^2}_{\text{Consumer Surplus}} + \underbrace{e_j \sum_{h\in[H]} \frac{1}{2}\alpha^{hh}_{jj}\left(s^{h*}_j\right)^2}_{\text{Producer Surplus}} \\
&\quad + \underbrace{\sum_{h\in[H]} \sum_{i\in[n]\setminus\{j\}} \tau^h_{ij} \left[\pi^h_i(Q^*) + c^h_{ij}(Q^*)\right] e_i Q^{h*}_{ij}}_{\text{Tariff Revenues}} - \underbrace{\sum_{l\in[L]} \sum_{h\in[H]} \sum_{i\in[n]} \alpha^{lh} r^h_{ij} \lambda^{l*}_j Q^{h*}_{ij}}_{\text{Government Subsidy Expenditure}},
\end{aligned}$$

where d^{h*}_j and s^{h*}_j refer to the equilibrium values of d^h_j and s^h_j, respectively. In other words, TES measures a country/group of countries' overall economic well-being, by combining the surpluses enjoyed by consumers and producers with government tariff revenues, minus the public spending required to meet nutritional constraints, to provide a clear snapshot of the total welfare generated by its economy.

Table 3 shows the annual supply, supply price, annual demand, demand price, and the total economic surplus of each country/group of countries at the equilibrium. Moreover, the last lines of the table show the total supply and demand, as well as the average supply and demand prices, calculated as weighted averages of prices relative to the corresponding quantities.

Scenario 2 is identical to Scenario 1 except that all the tariffs are equal to 0. This scenario models a condition of international free trade, characterized by the elimination of tariffs. While the food sector currently lacks a comprehensive zero-tariff agreement, a significant precedent exists in the pharmaceutical industry: the "Pharmaceutical Tariff Elimination Agreement (Zero for Zero)" entered into force in 1995 among 22 countries, eliminating tariffs on a multitude of pharmaceutical products. Table 4 shows the

[3] One should note that total demand is not equal to total supply due to deterioration.

[4] We set this restriction because people in these countries have diets strongly based on wheat products. In other countries, for cultural reasons, diets are based on other commodities such as rice. Therefore, the other countries/groups of countries have a minimum of 0 kcal provided by wheat. This approach may be extended to a hybrid framework that bridges static constraints – characterized by fixed, mandatory lower nutritional bounds – with dynamic demand modeling, wherein nutritional intake is endogenously determined by food prices, income levels, domestic consumer preferences, and utility maximization.

Table 3
Annual supply, supply price, annual demand, demand price, and TES of each country/group of countries in Scenario 1.

Country	Code	Supply (kt)	Supply price ($/t)	Demand (kt)	Demand price ($/t)	TES (M$)
Afghanistan	AFG	4999	294	5836	237	7.970
Argentina	ARG	16,565	219	6013	253	10.786
Australia	AUS	19,359	207	9146	261	19.694
Bangladesh	BGD	1211	339	7432	252	6.591
Brazil	BRA	6867	251	9431	275	2.787
Canada	CAN	38,874	192	6509	240	7.227
China	CHN	124,934	151	145,462	396	350.847
Germany	DEU	23,124	229	12,878	264	9.953
Algeria	DZA	3849	282	12,137	268	16.683
Egypt, Arab Rep.	EGY	7830	268	33,800	296	25.567
Spain	ESP	8146	254	9130	271	4.645
Ethiopia	ETH	5089	276	6654	261	27.534
France	FRA	31,117	217	8571	249	12.445
United Kingdom	GBR	10,085	249	9684	274	5.520
Hungary	HUN	5802	263	1857	223	2.301
Indonesia	IDN	0	340	10,026	271	10.384
India	IND	96,888	173	99,986	371	132.041
Iran, Islamic Rep.	IRN	14,494	249	19,576	284	156.561
Iraq	IRQ	4608	286	5339	239	29.818
Italy	ITA	6612	264	14,074	275	6.021
Japan	JPN	696	347	6339	316	13.310
Kazakhstan	KAZ	12,160	243	5245	238	4.950
Mexico	MEX	3217	278	6330	254	6.321
Morocco	MOR	2505	295	7825	278	8.081
Nigeria	NGA	0	297	6184	265	2.243
Pakistan	PAK	23,570	228	29,338	295	37.366
Philippines	PHL	0	340	6881	255	7.179
Poland	POL	13,574	243	7274	244	6.437
Romania	ROU	7068	261	2168	227	2.823
Russian Federation	RUS	89,991	166	22,293	282	28.283
Turkiye	TUR	12,232	252	19,179	294	12.859
Ukraine	UKR	26,883	210	6549	243	9.867
United States	USA	50,092	188	18,336	274	18.809
Uzbekistan	UZB	6217	278	7741	248	11.564
Rest of Asia-Oceania	XAO	552	335	1876	220	2.466
Rest of Central Asia	XCA	5062	284	6769	246	8.409
Rest of East Asia	XEA	1363	334	14,414	274	15.795
Rest of Middle East-North Africa	XME	6292	273	28,420	291	55.723
Central America and Caribbean	XOC	0	436	5777	248	11.993
Rest of Europe	XOE	40,830	212	22,465	275	16.516
Rest of South America	XOS	3314	248	8759	264	11.260
Rest of Sub-Saharan Africa	XSS	3479	288	20,496	295	24.269
Total		739,554		694,199		
Average price			200.36		313.44	

equilibrium quantities, prices, and total economic surpluses compared to Scenario 1. Scenario 2 reveals a mixed impact on supply and demand, with varying trends across different countries/groups of countries. Overall, aggregate global demand and supply are slightly higher than in Scenario 1 (the benchmark). Specifically, several "winners" emerge under this scenario, defined as countries/groups of countries that experience an increase in TES relative to the baseline. For instance, Russia's TES increases by nearly 5%, while two other major wheat producers, Canada and Ukraine, see growth of 2.59% and 2.74%, respectively. Japan, a major wheat importer that relies on foreign markets for over 80% of its needs, presents a unique case: its increase in TES is primarily driven by a nearly 20% plunge in demand prices. Crucially, Scenario 2 does not constitute a Pareto improvement, as certain nations experience a decline in TES values, such as Hungary (-1.03%) and Kazakhstan (-1.40%).

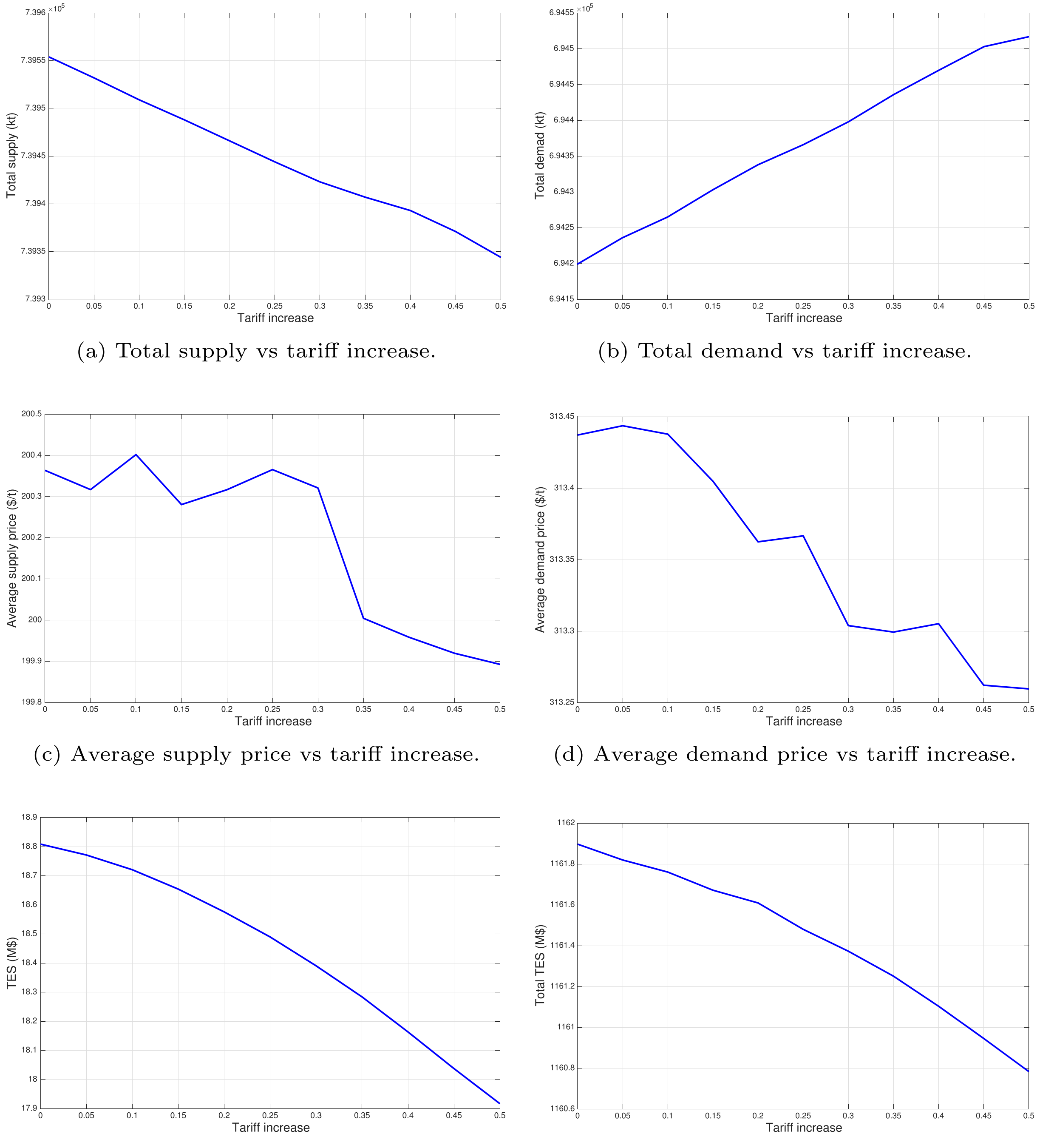
Scenario 3 is identical to Scenario 1 except for the tariffs imposed by the United States on its imports: each tariff increases by the same quantity p, with p varying from 0 to 0.5. Fig. 1 shows a sensitivity analysis of the total supply and demand, the average supply and demand prices, and the USA's TES and total TES with respect to the tariff increase. Table 5 shows the equilibrium quantities, prices, and total economic surpluses in the event of a 0.5 tariff increase, and the percentage differences compared to Scenario 1. Scenario 3 (which considers a 0.5 increase in US tariffs) shows a slight decline in global supply, accompanied by a marginal increase in global demand. In terms of TES, the United States experiences the most substantial contraction, with a sharp decline of nearly 5% compared to the Scenario 1 benchmark. Canada is also adversely affected, reporting a 0.43% decrease in TES. The remaining

Table 4
Annual supply, supply price, annual demand, demand price, and TES of each country/group of countries in Scenario 2. The percentage differences are relative to Scenario 1.

	Supply		Supply price		Demand		Demand price		TES	
Country	(kt)	Diff.	($/t)	Diff.	(kt)	Diff.	($/t)	Diff.	(M$)	Diff.
AFG	4966	−0.66%	285	−3.06%	5816	−0.34%	243	2.53%	7.894	−0.96%
ARG	16,469	−0.58%	216	−1.37%	5995	−0.30%	259	2.37%	10.694	−0.85%
AUS	19,402	0.22%	211	1.93%	8971	−1.91%	267	2.30%	19.719	0.13%
BGD	1204	−0.58%	330	−2.65%	7422	−0.13%	254	0.79%	6.556	−0.53%
BRA	6729	−2.01%	245	−2.39%	9553	1.29%	271	−1.45%	2.767	−0.70%
CAN	39,522	1.67%	198	3.13%	6257	−3.87%	246	2.50%	7.414	2.59%
CHN	125,240	0.24%	159	5.30%	145,822	0.25%	391	−1.26%	352.421	0.45%
DEU	23,085	−0.17%	227	−0.87%	13,135	2.00%	260	−1.52%	9.979	0.26%
DZA	3843	−0.16%	273	−3.19%	12,154	0.14%	267	−0.37%	16.650	−0.19%
EGY	7809	−0.27%	261	−2.61%	33,827	0.08%	295	−0.34%	25.542	−0.10%
ESP	8082	−0.79%	248	−2.36%	9533	4.41%	259	−4.43%	4.679	0.74%
ETH	5084	−0.10%	266	−3.62%	6660	0.09%	261	0.00%	27.456	−0.28%
FRA	31,153	0.12%	218	0.46%	8490	−0.95%	251	0.80%	12.450	0.04%
GBR	10,008	−0.76%	243	−2.41%	10,346	6.84%	255	−6.93%	5.602	1.47%
HUN	5785	−0.29%	261	−0.76%	1834	−1.24%	225	0.90%	2.278	−1.03%
IDN	0	0.00%	340	0.00%	10,019	−0.07%	273	0.74%	10.358	−0.25%
IND	97,067	0.18%	175	1.16%	101,043	1.06%	356	−4.04%	133.661	1.23%
IRN	14,489	−0.03%	242	−2.81%	19,639	0.32%	278	−2.11%	156.450	−0.07%
IRQ	4605	−0.07%	278	−2.80%	5287	−0.97%	248	3.77%	29.735	−0.28%
ITA	6565	−0.71%	258	−2.27%	14,401	2.32%	266	−3.27%	6.034	0.22%
JPN	695	−0.14%	338	−2.59%	6451	1.77%	255	−19.30%	13.692	2.87%
KAZ	12,119	−0.34%	240	−1.23%	5138	−2.04%	242	1.68%	4.880	−1.40%
MEX	3194	−0.71%	269	−3.24%	6314	−0.25%	258	1.57%	6.267	−0.85%
MOR	2499	−0.24%	284	−3.73%	8067	3.09%	260	−6.47%	8.190	1.35%
NGA	0	0.00%	297	0.00%	6220	0.58%	261	−1.51%	2.217	−1.18%
PAK	23,489	−0.34%	224	−1.75%	29,358	0.07%	294	−0.34%	37.264	−0.27%
PHL	0	0.00%	340	0.00%	6869	−0.17%	259	1.57%	7.151	−0.39%
POL	13,515	−0.43%	240	−1.23%	7241	−0.45%	246	0.82%	6.380	−0.88%
ROU	7033	−0.50%	257	−1.53%	2167	−0.05%	227	0.00%	2.797	−0.92%
RUS	92,197	2.45%	180	8.43%	22,265	−0.13%	282	0.00%	29.618	4.72%
TUR	12,184	−0.39%	247	−1.98%	20,004	4.30%	277	−5.78%	13.128	2.09%
UKR	27,335	1.68%	222	5.71%	6463	−1.31%	245	0.82%	10.138	2.74%
USA	50,342	0.50%	191	1.60%	18,337	0.01%	274	0.00%	18.897	0.47%
UZB	6201	−0.26%	270	−2.88%	7626	−1.49%	254	2.42%	11.469	−0.82%
XAO	551	−0.18%	328	−2.09%	1867	−0.48%	227	3.18%	2.446	−0.80%
XCA	5046	−0.32%	275	−3.17%	6620	−2.20%	253	2.85%	8.317	−1.09%
XEA	1361	−0.15%	327	−2.10%	14,373	−0.28%	279	1.82%	15.715	−0.50%
XME	6291	−0.02%	271	−0.73%	28,430	0.04%	290	−0.34%	55.720	−0.01%
XOC	0	0.00%	436	0.00%	5767	−0.17%	255	2.82%	11.954	−0.33%
XOE	40,789	−0.10%	211	−0.47%	22,264	−0.89%	276	0.36%	16.448	−0.41%
XOS	3404	2.72%	267	7.66%	8730	−0.33%	272	3.03%	11.260	0.00%
XSS	3477	−0.06%	283	−1.74%	20,617	0.59%	291	−1.36%	24.207	−0.25%
Total	742,833	0.44%			697,392	0.46%				
Avg price			203.20	1.41%			308.80	−1.48%		

countries/groups of countries analyzed exhibit minimal impact, with variations restricted to a range of ±0.25% relative to Scenario 1. Moreover, the sensitivity analysis (from 0 to 0.5 tariff increase in US) indicates that further tariff increases lead to a contraction in aggregate supply; conversely, aggregate demand expands as tariffs rise. Notably, the sensitivity analysis reveals that higher US tariffs strictly contract both US and total TES.

Scenario 4 is identical to Scenario 1 except for the tariffs imposed by China on its imports: each tariff increases by the same quantity p, with p varying from 0 to 0.2. Fig. 2 shows a sensitivity analysis of the total supply and demand, the average supply and demand prices, and the Chinese TES and total TES with respect to the tariff increase. Table 6 shows the equilibrium quantities, prices, and total economic surpluses in the event of a 0.2 tariff increase, and the percentage differences compared to Scenario 1. Scenario 4 (assuming a 20% increase in Chinese tariffs) shows total supply and total demand declining by 0.31% and 0.24%, respectively. In terms of TES, China experiences the most pronounced contraction at −1.47%. Several other nations also register TES declines exceeding 1%, namely Canada (−1.19%), Romania (−1.16%), and Hungary (−1.13%). Regarding price dynamics, China faces a substantial double-digit surge of 16.67% in its domestic demand price of wheat, whereas price variations across all other analyzed countries/groups of countries are either negative or negligible. This extraordinary spike in Chinese wheat prices could potentially act as a catalyst for domestic inflation. Furthermore, the sensitivity analysis (spanning tariff increases from 0 to 0.2) indicates a



(a) Total supply vs tariff increase.

(b) Total demand vs tariff increase.

(c) Average supply price vs tariff increase.

(d) Average demand price vs tariff increase.

(e) USA's TES vs tariff increase.

(f) Total TES vs tariff increase.

Fig. 1. Scenario 3: total production (a), total demand (b), average supply price (c), average demand price (d), USA's TES (e), and total TES (f) as functions of the USA's tariff increase.

simultaneous decline in both aggregate supply and aggregate demand. Crucially, the sensitivity analysis shows that increasing the Chinese tariffs drives a strict contraction in both Chinese and total TES. Consistent with Scenario 3, these results underscore that countries imposing unilateral tariffs ultimately suffer the most severe damages.

Scenario 5 is identical to Scenario 1, with the addition of the constraints that flows from Russia to European countries (and vice versa) are all equal to 0. Indeed, this Scenario 5 – regarding the current geopolitical situation – postulates a severe deterioration

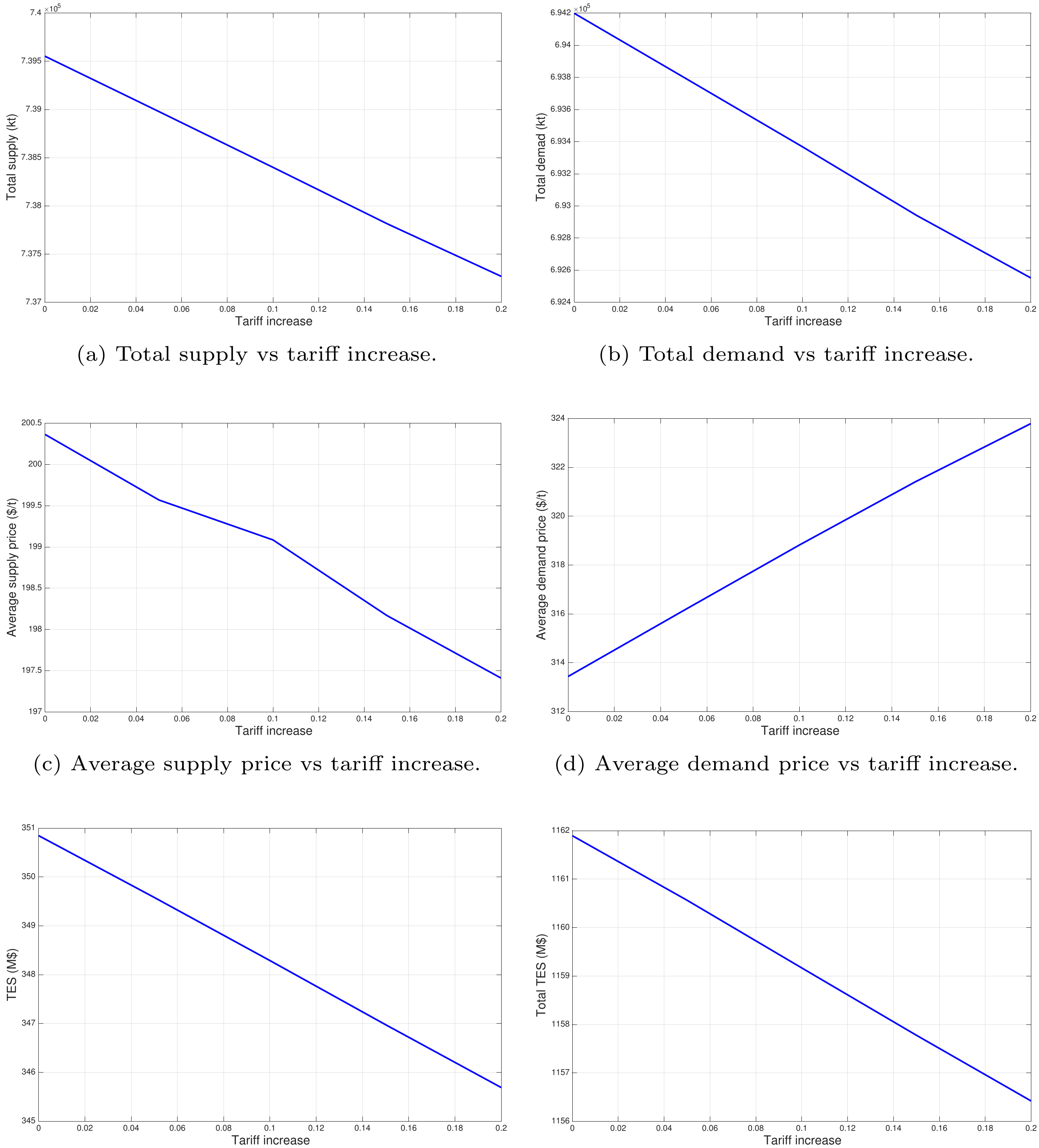
Table 5
Annual supply, supply price, annual demand, demand price, and TES of each country/group of countries in Scenario 3 in the event of a 0.5 tariff increase. The percentage differences are relative to Scenario 1.

	Supply		Supply price		Demand		Demand price		TES	
Country	(kt)	Diff.	($/t)	Diff.	(kt)	Diff.	($/t)	Diff.	(M$)	Diff.
AFG	4997	−0.04%	293	−0.34%	5839	0.05%	237	0.00%	7.972	0.02%
ARG	16,545	−0.12%	219	0.00%	6012	−0.02%	253	0.00%	10.773	−0.12%
AUS	19,347	−0.06%	206	−0.48%	9149	0.03%	261	0.00%	19.672	−0.11%
BGD	1211	0.00%	339	0.00%	7436	0.05%	251	−0.40%	6.597	0.09%
BRA	6862	−0.07%	251	0.00%	9440	0.10%	275	0.00%	2.787	0.03%
CAN	38,786	−0.23%	192	0.00%	6502	−0.11%	240	0.00%	7.195	−0.43%
CHN	124,912	−0.02%	150	−0.66%	145,481	0.01%	396	0.00%	350.820	−0.01%
DEU	23,115	−0.04%	228	−0.44%	12,888	0.08%	264	0.00%	9.948	−0.04%
DZA	3849	0.00%	282	0.00%	12,143	0.05%	268	0.00%	16.688	0.03%
EGY	7829	−0.01%	268	0.00%	33,812	0.04%	296	0.00%	25.577	0.04%
ESP	8143	−0.04%	254	0.00%	9135	0.05%	271	0.00%	4.646	0.01%
ETH	5089	0.00%	276	0.00%	6657	0.05%	261	0.00%	27.536	0.01%
FRA	31,080	−0.12%	216	−0.46%	8576	0.06%	249	0.00%	12.419	−0.21%
GBR	10,080	−0.05%	248	−0.40%	9685	0.01%	274	0.00%	5.518	−0.04%
HUN	5799	−0.05%	263	0.00%	1864	0.38%	222	−0.45%	2.301	−0.03%
IDN	0	0.00%	340	0.00%	10,029	0.03%	271	0.00%	10.390	0.05%
IND	96,839	−0.05%	172	−0.58%	100,003	0.02%	371	0.00%	132.004	−0.03%
IRN	14,493	−0.01%	248	−0.40%	19,581	0.03%	283	−0.35%	156.551	−0.01%
IRQ	4608	0.00%	286	0.00%	5343	0.07%	238	−0.42%	29.822	0.01%
ITA	6608	−0.06%	264	0.00%	14,084	0.07%	275	0.00%	6.022	0.02%
JPN	696	0.00%	347	0.00%	6339	0.00%	317	0.32%	13.309	−0.01%
KAZ	12,156	−0.03%	243	0.00%	5260	0.29%	237	−0.42%	4.950	0.00%
MEX	3216	−0.03%	278	0.00%	6330	0.00%	254	0.00%	6.320	−0.02%
MOR	2504	−0.04%	293	−0.68%	7835	0.13%	278	0.00%	8.082	0.02%
NGA	0	0.00%	297	0.00%	6188	0.06%	264	−0.38%	2.247	0.16%
PAK	23,565	−0.02%	228	0.00%	29,346	0.03%	295	0.00%	37.371	0.01%
PHL	0	0.00%	340	0.00%	6883	0.03%	255	0.00%	7.183	0.05%
POL	13,570	−0.03%	243	0.00%	7276	0.03%	244	0.00%	6.435	−0.03%
ROU	7064	−0.06%	260	−0.38%	2176	0.37%	226	−0.44%	2.822	−0.05%
RUS	89,898	−0.10%	165	−0.60%	22,350	0.26%	282	0.00%	28.233	−0.18%
TUR	12,229	−0.02%	252	0.00%	19,193	0.07%	294	0.00%	12.862	0.02%
UKR	26,843	−0.15%	209	−0.48%	6575	0.40%	242	−0.41%	9.844	−0.23%
USA	50,351	0.52%	191	1.60%	18,336	0.00%	274	0.00%	17.917	−4.74%
UZB	6217	0.00%	278	0.00%	7741	0.00%	248	0.00%	11.564	0.00%
XAO	552	0.00%	335	0.00%	1876	0.00%	220	0.00%	2.466	0.00%
XCA	5062	0.00%	284	0.00%	6773	0.06%	246	0.00%	8.410	0.01%
XEA	1363	0.00%	334	0.00%	14,414	0.00%	274	0.00%	15.794	0.00%
XME	6292	0.00%	273	0.00%	28,426	0.02%	290	−0.34%	55.730	0.01%
XOC	0	0.00%	436	0.00%	5775	−0.03%	249	0.40%	11.987	−0.05%
XOE	40,778	−0.13%	211	−0.47%	22,499	0.15%	274	−0.36%	16.487	−0.18%
XOS	3313	−0.03%	247	−0.40%	8758	−0.01%	264	0.00%	11.257	−0.02%
XSS	3479	0.00%	288	0.00%	20,509	0.06%	294	−0.34%	24.276	0.03%
Total	739,344	−0.03%			694,517	0.05%				
Avg price			199.89	−0.24%			313.26	−0.06%		

of relations between the Russian Federation and the EU, leading to the imposition of a comprehensive trade ban that encompasses agricultural products. The corresponding results are shown in Table 7. Scenario 5 exhibits a general decline in total demand, total supply, and average demand and supply prices. Russia emerges as the most severely affected nation, with its TES contracting by 3.76% relative to the benchmark. Its domestic prices display divergent dynamics: the supply price plummets by 6.63%, whereas the demand price increases by 2.13%. Similarly, Ukraine suffers a 1.51% reduction in its TES value. Among the three major EU economies analyzed, Germany registers a marginal increase of 0.02%, while France and Italy experience contractions in TES value of 0.50% and 0.80%, respectively. Given these results, we can state that on the EU side, countries show tiny variations compared to Scenario 1. This suggests that a trade ban would pose a severe problem primarily for Russia.

Finally, Scenario 6 is identical to Scenario 1, except for spoilage factors, which vary from 0.75 to 1 according to the following definition:

$$r_{ij}^h = 1 - 0.25\,\frac{b_{ij}^h}{\max\limits_{i,j\in[n]} b_{ij}^h}.$$



(a) Total supply vs tariff increase.

(b) Total demand vs tariff increase.

(c) Average supply price vs tariff increase.

(d) Average demand price vs tariff increase.

(e) China's TES vs tariff increase.

(f) Total TES vs tariff increase.

Fig. 2. Scenario 4: total production (a), total demand (b), average supply price (c), average demand price (d), China's TES (e), and total TES (f) as functions of the China's tariff increase.

This scenario investigates the robustness of the results if the deterioration rate is higher than predicted in Scenario 1. The formula above takes into account what is found in [26], where a higher level of deterioration was estimated in the case of road wheat transportation.[5] The corresponding results are shown in Table 8. In this final scenario, which models an exogenous shock to the

[5] They compute a loss of 0.479 kg per km in road wheat transportation, for an initial load of 38,000 kg. Linearly extrapolating this rate – with all necessary caveats – over a hypothetical trade distance of 20,000 km (about one half of the equatorial circumference of the Earth) results in a total volume loss of roughly 25%, whereas modeling this dynamically to account for the continuously diminishing cargo weight reduces the projected loss to approximately 22%. It is important to note that

Table 6
Annual supply, supply price, annual demand, demand price, and TES of each country/group of countries in Scenario 4 in the event of a 0.2% tariff increase. The percentage differences are relative to Scenario 1.

	Supply		Supply price		Demand		Demand price		TES	
Country	(kt)	Diff.	($/t)	Diff.	(kt)	Diff.	($/t)	Diff.	(M$)	Diff.
AFG	4973	−0.52%	287	−2.38%	5844	0.14%	235	−0.84%	7.945	−0.31%
ARG	16,468	−0.59%	216	−1.37%	6020	0.12%	250	−1.19%	10.743	−0.39%
AUS	19,327	−0.17%	204	−1.45%	9207	0.67%	259	−0.77%	19.654	−0.21%
BGD	1206	−0.41%	333	−1.77%	7440	0.11%	250	−0.79%	6.596	0.07%
BRA	6763	−1.51%	246	−1.99%	9508	0.82%	272	−1.09%	2.778	−0.29%
CAN	38,590	−0.73%	190	−1.04%	6585	1.17%	238	−0.83%	7.141	−1.19%
CHN	124,915	−0.02%	150	−0.66%	140,934	−3.11%	462	16.67%	345.688	−1.47%
DEU	23,029	−0.41%	226	−1.31%	13,021	1.11%	262	−0.76%	9.915	−0.37%
DZA	3845	−0.10%	276	−2.13%	12,173	0.30%	265	−1.12%	16.692	0.06%
EGY	7814	−0.20%	263	−1.87%	33,927	0.38%	293	−1.01%	25.650	0.32%
ESP	8107	−0.48%	250	−1.57%	9216	0.94%	268	−1.11%	4.634	−0.25%
ETH	5086	−0.06%	270	−2.17%	6676	0.33%	259	−0.77%	27.519	−0.05%
FRA	30,993	−0.40%	214	−1.38%	8651	0.93%	246	−1.20%	12.375	−0.57%
GBR	10,039	−0.46%	245	−1.61%	9778	0.97%	271	−1.09%	5.505	−0.27%
HUN	5762	−0.69%	258	−1.90%	1873	0.86%	221	−0.90%	2.275	−1.13%
IDN	0	0.00%	340	0.00%	10,039	0.13%	269	−0.74%	10.411	0.26%
IND	96,706	−0.19%	170	−1.73%	100,334	0.35%	366	−1.35%	132.288	0.19%
IRN	14,491	−0.02%	245	−1.61%	19,613	0.19%	281	−1.06%	156.560	0.00%
IRQ	4605	−0.07%	278	−2.80%	5351	0.22%	237	−0.84%	29.794	−0.08%
ITA	6578	−0.51%	260	−1.52%	14,182	0.77%	272	−1.09%	6.026	0.08%
JPN	695	−0.14%	338	−2.59%	6344	0.08%	314	−0.63%	13.320	0.07%
KAZ	12,101	−0.49%	239	−1.65%	5286	0.78%	236	−0.84%	4.914	−0.72%
MEX	3199	−0.56%	271	−2.52%	6340	0.16%	252	−0.79%	6.313	−0.13%
MOR	2501	−0.16%	287	−2.71%	7861	0.46%	276	−0.72%	8.084	0.05%
NGA	0	0.00%	297	0.00%	6208	0.39%	262	−1.13%	2.260	0.72%
PAK	23,510	−0.25%	225	−1.32%	29,399	0.21%	292	−1.02%	37.375	0.02%
PHL	0	0.00%	340	0.00%	6887	0.09%	253	−0.78%	7.192	0.18%
POL	13,512	−0.46%	240	−1.23%	7321	0.65%	241	−1.23%	6.410	−0.42%
ROU	7018	−0.71%	256	−1.92%	2190	1.01%	225	−0.88%	2.790	−1.16%
RUS	89,675	−0.35%	164	−1.20%	22,734	1.98%	280	−0.71%	28.140	−0.51%
TUR	12,198	−0.28%	248	−1.59%	19,345	0.87%	291	−1.02%	12.878	0.14%
UKR	26,773	−0.41%	207	−1.43%	6621	1.10%	241	−0.82%	9.805	−0.63%
USA	49,899	−0.39%	185	−1.60%	18,337	0.01%	274	0.00%	18.732	−0.41%
UZB	6203	−0.23%	271	−2.52%	7788	0.61%	246	−0.81%	11.536	−0.24%
XAO	551	−0.18%	328	−2.09%	1879	0.16%	218	−0.91%	2.465	−0.04%
XCA	5049	−0.26%	277	−2.46%	6816	0.69%	244	−0.81%	8.385	−0.28%
XEA	1360	−0.22%	323	−3.29%	14,434	0.14%	271	−1.09%	15.815	0.13%
XME	6289	−0.05%	266	−2.56%	28,515	0.33%	287	−1.37%	55.780	0.10%
XOC	0	0.00%	436	0.00%	5781	0.07%	245	−1.21%	12.008	0.13%
XOE	40,664	−0.41%	209	−1.42%	22,735	1.20%	272	−1.09%	16.454	−0.38%
XOS	3294	−0.60%	243	−2.02%	8767	0.09%	262	−0.76%	11.265	0.04%
XSS	3477	−0.06%	283	−1.74%	20,592	0.47%	292	−1.02%	24.312	0.18%
Total	737,269	−0.31%			692,552	−0.24%				
Avg price			197.41	−1.47%			323.79	3.30%		

perishability rate, we observe a substantial 19.19% increase in average global demand prices. While the United States remains unaffected, Japan and China experience the most pronounced surges at 24.68% and 23.48%, respectively. Consequently, aggregate global demand for wheat contracts by 7.28%, whereas aggregate supply expands marginally by 1.57%. The majority of the analyzed countries/groups of countries exhibit a marked reduction in TES, characterized by sharp double-digit contractions in Nigeria (−13.84%) and Brazil (−10.08%). Conversely, a few nations counterintuitively benefit from the higher perishability rate, most notably Romania (+4.92%), Hungary (+4.08%), Kazakhstan (+3.79%), and Ukraine (+3.45%). This finding indicates that not only tariffs or trade wars may have negative impacts on price and quantity, but transportation costs and the capacity to store the commodity (e.g., wheat) must also be considered as critical factors influencing international trade.

these are only gross theoretical estimates, as they assume simplified spillage dynamics over an extreme distance that would otherwise necessitate the combination of various transport modalities.

Table 7
Annual supply, supply price, annual demand, demand price, and TES of each country/group of countries in Scenario 5. The percentage differences are relative to Scenario 1.

	Supply		Supply price		Demand		Demand price		TES	
Country	(kt)	Diff.	($/t)	Diff.	(kt)	Diff.	($/t)	Diff.	(M$)	Diff.
AFG	4996	−0.06%	293	−0.34%	5851	0.26%	234	−1.27%	7.989	0.23%
ARG	16,547	−0.11%	219	0.00%	6018	0.08%	251	−0.79%	10.787	0.01%
AUS	19,354	−0.03%	206	−0.48%	9201	0.60%	259	−0.77%	19.702	0.04%
BGD	1211	0.00%	339	0.00%	7452	0.27%	248	−1.59%	6.621	0.45%
BRA	6847	−0.29%	250	−0.40%	9482	0.54%	273	−0.73%	2.797	0.36%
CAN	38,832	−0.11%	192	0.00%	6596	1.34%	238	−0.83%	7.226	−0.01%
CHN	124,915	−0.02%	150	−0.66%	145,517	0.04%	395	−0.25%	350.915	0.02%
DEU	23,144	0.09%	229	0.00%	12,837	−0.32%	265	0.38%	9.955	0.02%
DZA	3849	0.00%	282	0.00%	12,163	0.21%	266	−0.75%	16.703	0.12%
EGY	7828	−0.03%	267	−0.37%	33,832	0.09%	295	−0.34%	25.593	0.10%
ESP	8144	−0.02%	254	0.00%	9112	−0.20%	271	0.00%	4.638	−0.15%
ETH	5089	0.00%	276	0.00%	6676	0.33%	259	−0.77%	27.551	0.06%
FRA	31,097	−0.06%	217	0.00%	8372	−2.32%	254	2.01%	12.383	−0.50%
GBR	10,092	0.07%	249	0.00%	9619	−0.67%	276	0.73%	5.508	−0.23%
HUN	5800	−0.03%	263	0.00%	1782	−4.04%	231	3.59%	2.291	−0.46%
IDN	0	0.00%	340	0.00%	10,037	0.11%	269	−0.74%	10.407	0.22%
IND	96,845	−0.04%	172	−0.58%	100,037	0.05%	370	−0.27%	132.084	0.03%
IRN	14,495	0.01%	250	0.40%	19,591	0.08%	282	−0.70%	156.613	0.03%
IRQ	4608	0.00%	286	0.00%	5361	0.41%	235	−1.67%	29.839	0.07%
ITA	6611	−0.02%	264	0.00%	13,972	−0.72%	278	1.09%	5.972	−0.80%
JPN	696	0.00%	347	0.00%	6338	−0.02%	317	0.32%	13.308	−0.02%
KAZ	12,164	0.03%	243	0.00%	5339	1.79%	234	−1.68%	4.969	0.39%
MEX	3215	−0.06%	277	−0.36%	6344	0.22%	252	−0.79%	6.339	0.29%
MOR	2504	−0.04%	293	−0.68%	7857	0.41%	276	−0.72%	8.103	0.27%
NGA	0	0.00%	297	0.00%	6203	0.31%	263	−0.75%	2.257	0.58%
PAK	23,577	0.03%	229	0.44%	29,358	0.07%	294	−0.34%	37.404	0.10%
PHL	0	0.00%	340	0.00%	6890	0.13%	252	−1.18%	7.200	0.29%
POL	13,593	0.14%	244	0.41%	7154	−1.65%	250	2.46%	6.404	−0.51%
ROU	7060	−0.11%	260	−0.38%	2184	0.74%	226	−0.44%	2.820	−0.10%
RUS	88,447	−1.72%	155	−6.63%	20,956	−6.00%	288	2.13%	27.218	−3.76%
TUR	12,225	−0.06%	252	0.00%	19,193	0.07%	294	0.00%	12.855	−0.03%
UKR	26,754	−0.48%	207	−1.43%	6362	−2.86%	248	2.06%	9.718	−1.51%
USA	50,070	−0.04%	188	0.00%	18,338	0.01%	274	0.00%	18.827	0.10%
UZB	6217	0.00%	278	0.00%	7797	0.72%	246	−0.81%	11.585	0.18%
XAO	552	0.00%	335	0.00%	1880	0.21%	217	−1.36%	2.471	0.23%
XCA	5061	−0.02%	284	0.00%	6835	0.98%	243	−1.22%	8.426	0.21%
XEA	1363	0.00%	334	0.00%	14,427	0.09%	272	−0.73%	15.817	0.14%
XME	6292	0.00%	273	0.00%	28,445	0.09%	290	−0.34%	55.748	0.05%
XOC	0	0.00%	436	0.00%	5781	0.07%	245	−1.21%	12.010	0.14%
XOE	40,793	−0.09%	212	0.00%	21,905	−2.49%	280	1.82%	16.382	−0.82%
XOS	3308	−0.18%	246	−0.81%	8766	0.08%	262	−0.76%	11.273	0.12%
XSS	3478	−0.03%	286	−0.69%	20,537	0.20%	294	−0.34%	24.288	0.08%
Total	737,677	−0.25%			692,397	−0.26%				
Avg price			198.68	−0.84%			313.07	−0.12%		

5. Conclusion and further research perspectives

This paper has extended the multicommodity international trade network equilibrium framework to account explicitly for nutritional constraints, *ad valorem* tariffs, and losses due to perishability. Within this setting, the equilibrium problem was formulated as a variational inequality, and sufficient conditions ensuring the strong monotonicity of the associated operator were derived. These results guarantee the existence and uniqueness of the equilibrium, providing a theoretical foundation for algorithmic convergence. The numerical experiments, calibrated with real-world data from the global wheat market, illustrated how tariffs and perishability jointly affect international trade flows, equilibrium prices, and food security outcomes. In particular, the scenarios examined highlight the trade-offs policymakers face when balancing protectionist measures, transportation inefficiencies, and nutritional standards.

While our framework accommodates multiple food commodities, expanding the numerical experiments beyond wheat to include cross-commodity substitutability is a compelling direction for future research. Calibrating such an empirical application requires careful consideration, as accurately capturing non-diagonal cross-price effects without economic separability significantly increases data demands. Moreover, outside the food sector, the underlying mathematical structure can easily adapt to other domains. For instance, a future variant could model the automotive market by replacing nutritional requirements with environmental constraints – such as fleet-wide emission mandates – applied to vehicles with varying levels of greenness.

Beyond the specific insights gained from the empirical application investigated in the paper, the underlying mathematical framework possesses structural properties that ensure broader generalizability and robustness. First, we emphasize that network symmetry

Table 8
Annual supply, supply price, annual demand, demand price, and TES of each country/group of countries in Scenario 6. The percentage differences are relative to Scenario 1.

	Supply		Supply price		Demand		Demand price		TES	
Country	(kt)	Diff.	($/t)	Diff.	(kt)	Diff.	($/t)	Diff.	(M$)	Diff.
AFG	5155	3.12%	338	14.97%	5705	−2.24%	272	14.77%	7.996	0.32%
ARG	16,907	2.06%	231	5.48%	5871	−2.36%	299	18.18%	10.711	−0.69%
AUS	19,437	0.40%	214	3.38%	8019	−12.32%	301	15.33%	19.500	−0.98%
BGD	1255	3.63%	392	15.63%	7263	−2.27%	287	13.89%	6.402	−2.87%
BRA	7277	5.97%	271	7.97%	8144	−13.65%	322	17.09%	2.506	−10.08%
CAN	39,663	2.03%	199	3.65%	5395	−17.11%	266	10.83%	7.343	1.61%
CHN	125,234	0.24%	158	4.64%	139,077	−4.39%	489	23.48%	338.567	−3.50%
DEU	23,815	2.99%	251	9.61%	10,983	−14.72%	297	12.50%	10.071	1.19%
DZA	3866	0.44%	308	9.22%	11,531	−4.99%	313	16.79%	16.255	−2.56%
EGY	7950	1.53%	305	13.81%	31,882	−5.67%	350	18.24%	24.089	−5.78%
ESP	8419	3.35%	282	11.02%	7866	−13.84%	309	14.02%	4.520	−2.70%
ETH	5105	0.31%	307	11.23%	6197	−6.87%	310	18.77%	27.381	−0.55%
FRA	31,933	2.62%	236	8.76%	7396	−13.71%	283	13.65%	12.778	2.67%
GBR	10,362	2.75%	271	8.84%	8552	−11.69%	306	11.68%	5.414	−1.92%
HUN	6014	3.65%	290	10.27%	1574	−15.24%	256	14.80%	2.395	4.08%
IDN	0	0.00%	340	0.00%	9772	−2.53%	324	19.56%	9.862	−5.03%
IND	97,655	0.79%	183	5.78%	94,377	−5.61%	449	21.02%	125.489	−4.96%
IRN	14,519	0.17%	282	13.25%	18,973	−3.08%	335	17.96%	156.083	−0.31%
IRQ	4622	0.30%	322	12.59%	5120	−4.10%	277	15.90%	29.785	−0.11%
ITA	6807	2.95%	293	10.98%	12,707	−9.71%	313	13.82%	5.668	−5.86%
JPN	702	0.86%	405	16.71%	6196	−2.26%	394	24.68%	12.994	−2.38%
KAZ	12,606	3.67%	269	10.70%	4444	−15.27%	267	12.18%	5.138	3.79%
MEX	3256	1.21%	293	5.40%	6120	−3.32%	300	18.11%	6.085	−3.73%
MOR	2519	0.56%	320	8.47%	7222	−7.71%	325	16.91%	7.792	−3.58%
NGA	0	0.00%	297	0.00%	5721	−7.49%	318	20.00%	1.933	−13.84%
PAK	24,063	2.09%	256	12.28%	28,263	−3.66%	349	18.31%	36.486	−2.36%
PHL	0	0.00%	340	0.00%	6738	−2.08%	298	16.86%	6.884	−4.11%
POL	14,011	3.22%	267	9.88%	6675	−8.23%	276	13.11%	6.537	1.56%
ROU	7339	3.83%	289	10.73%	1802	−16.88%	260	14.54%	2.962	4.92%
RUS	91,466	1.64%	176	6.02%	15,839	−28.95%	312	10.64%	28.602	1.13%
TUR	12,524	2.39%	287	13.89%	16,791	−12.45%	345	17.35%	12.384	−3.70%
UKR	27,612	2.71%	229	9.05%	5522	−15.68%	271	11.52%	10.207	3.45%
USA	50,517	0.85%	193	2.66%	18,337	0.01%	274	0.00%	18.093	−3.81%
UZB	6297	1.29%	320	15.11%	6997	−9.61%	284	14.52%	11.561	−0.02%
XAO	558	1.09%	379	13.13%	1825	−2.72%	259	17.73%	2.419	−1.91%
XCA	5126	1.26%	321	13.03%	5960	−11.95%	283	15.04%	8.366	−0.51%
XEA	1379	1.17%	392	17.37%	13,969	−3.09%	327	19.34%	15.116	−4.30%
XME	6311	0.30%	312	14.29%	26,926	−5.26%	344	18.21%	54.501	−2.19%
XOC	0	0.00%	436	0.00%	5712	−1.13%	295	18.95%	11.723	−2.25%
XOE	41,972	2.80%	232	9.43%	18,839	−16.14%	306	11.27%	16.672	0.94%
XOS	3450	4.10%	277	11.69%	8568	−2.18%	314	18.94%	10.927	−2.96%
XSS	3490	0.32%	320	11.11%	18,757	−8.48%	352	19.32%	23.260	−4.16%
Total	751,197	1.57%			643,627	−7.28%				
Avg price			216.24	7.92%			373.58	19.19%		

is not a strict requirement of our model, which is highlighted by the fact that the Jacobian matrices ∇F_1, ∇F_2, and ∇F_3 used in the proof of Theorem 3 in general are not symmetric (though their symmetric parts are used in that proof). Moreover, although the framework allows each country/group of countries to act simultaneously as both a supply and a demand node, the mathematical equilibrium strictly precludes simultaneous cross-trade (i.e., the two-way exchange of the same homogeneous commodity) between any two given countries/groups of countries. Second, our formulation can easily accommodate networks where the number of supply and demand nodes differs. A country/group of countries can be functionally removed as a supply or demand node for a specific product simply by assuming a complete loss of the commodity during transit. Specifically, one can set the survival fraction to zero for all outgoing shipments from a country/group of countries that do not act as a supplier, or for all incoming shipments to a country/group of countries that do not act as a consumer.

Finally, further extensions, such as incorporating parallel directed arcs within any pair of countries/groups of countries (e.g., to model distinct transportation modalities individually rather than in aggregate) or allowing for multi-hop transit routes (e.g., leveraging secondary paths for legal tariff mitigation via route diversion), are entirely feasible. In such extensions, the competitive nature of the agents ensures that any parallel routes actively used between the same origin and destination must reach an equilibrium state where their generalized transportation costs are identical. However, introducing these structural complexities may alter the sufficient conditions required for the strong monotonicity of the operators associated with the corresponding variational inequality problems.

Future research may address several other major extensions of the present work. First, the framework can be generalized to a dynamic, multi-period setting, allowing for the modeling of temporal perishability, that is, the progressive deterioration of commodities over time during storage or successive trading periods. Second, the model could be enriched to incorporate exchange rate volatility and additional forms of trade policy intervention. Finally, an interesting but technically challenging direction is to make trade policies endogenous within the network model by coupling the agents' equilibrium conditions with a policymaker's optimization problem that selects tariffs or subsidies to achieve welfare or food-security objectives. This would lead to a bilevel formulation, or equivalently, a Mathematical Program with Equilibrium Constraints (MPEC), where the upper level represents the policy design problem and the lower level captures the trade equilibrium.

CRediT authorship contribution statement

Francesco Biancalani: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Giorgio Gnecco:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Mauro Passacantando:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Fabio Raciti:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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